Temporal Perspective

A Logical Analysis of Temporal Reference in English

Paul Needham

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To my parents

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Preface

This is an investigation into the logic of temporal reference. The term 'tense logic' is deliberately avoided here since, although tenses receive by far the most attention in what follows, this in no way undermines the importance of alternative means available in natural language for expressing reference to time. Those logicians interested in "taking tenses seriously" have done it so seriously as to exclude virtually all else, and I shall endeavour to redress the balance.

The book is both critical and constructive. Critical, in that it is opposed to the basic tenet in the writings of Arthur Prior—the founder of modern tense logic—that tenses behave essentially very much like *propositional* modal operators. The mainspring of the analogy is Prior's view that the structure of complex tenses comprises an *iterated* string of tense operators. That is to say, Prior proposed that it is possible to iterate tenses in the same way that propositional modal operators are iterated in expressions like 'It is possible that it is necessary that it might be the case that'. This view derives from that of McTaggart (as does the imbalance mentioned in the previous paragraph), and so in trying to begin afresh as it were, I have taken McTaggart's famous argument as my starting point. The constructive aspect lies in developing the expressive powers of a first-order logic and demonstrating the applicability of its greater flexibility over conventional tense logic.

Such proofs as there are were included more for the sake of completeness than originality. The important results are theorems 5 and 10. Theorem 5 embodies within the formal system what I take to be the crucial principle, that the point of view of the speaker dominates all subordinate contexts. The standard prenex normal form theorem, theorem 10, lays the foundation for the definition of a tense.

I have benefitted from discussions of various parts of the material presented here with Professor Stig Kanger, Dr. Tim Moore, Mr. Fred Benenson and Dr. Hans Kamp, but particularly from Dr. Ingmar Pörn who read and helpfully commented upon the whole draft manuscript. The influence of a former teacher of mine, Dr. Peter Röper, must also be acknowledged and the notion of a comparison developed in Chapter IV derives from a suggestion of his. Christine Jobes made a marvellous job of typing a difficult manuscript and Mick Green was a great help preparing the index and checking the typescript. Finally, without the help of Margareta Sjöberg and Jan Odelstad I doubt whether the book would have passed the typescript stage.

Birmingham, February 1975 Paul Needham

Chapter I

McTaggart's Proof of the Unreality of Time

1. Introduction

Much of the recent work on that aspect of propositions concerned with reference to positions in time has been directly provoked by McTaggart's famous proof of the unreality of time. For in spite of the fact that few, if any, would accept his argument, his views on the nature of tenses have, on the whole, been adopted in what is now widely understood as orthodox tense logic. Thus Prior, on the first page of *Past, Present and Future*, says: "McTaggart. . . drew attention to a body of facts about time which we shall be adverting to frequently in what follows. Indeed, one could say that there is a tense-logic itself in McTaggart. . ."[1] Moreover, the views of many philosophers who oppose the recent school of tense logicians, whilst not so accurately portrayed by McTaggart, can be seen as one possible response to his argument.

Since the appearance of McTaggart's argument it has become customary in some circles to draw a rigid distinction between tensed statements and those statements which refer to positions in time without the use of tenses. Along with the drawing of this distinction there has arisen a dispute between the so-called A-theorists, who maintain that tenseless statements can be reduced to tensed statements which, they claim, give an adequate account of positions in time, and the B-theorists, who maintain that tenseless statements have the privileged status and that tensed statements can be reduced to these. The B-theorists want to emphasise a strong analogy between space and time, whereas the A-theorists, following McTaggart, insist that there is a significant disanalogy which is reflected in their program of expressing what is conveyed by tenseless statements in terms of tensed statements. The precise points of disanalogy have been variously expressed, but I think it fair to say that A-theorists feel there is a significant difference between past and future which is not reflected by a pair of spatial regions to the left and right of any given point. Some philosophers have also talked of a 'movement' of the present from the past towards the future which is thought to have no analogue in space. Whether there is any substance in these claims

[1] Arthur Prior, *Past, Present and Future* (Oxford, 1967).

about differences between past and future, and the 'movement of the present', which would justify abandoning the analogy, I shall not consider in this essay. More important for my purposes is the articulation of the logical relations between tenseless statements of the kind 'M occurs at time t' and 'M is earlier than N', and purely tensed propositions, and the question of the reducibility of tensed to tenseless statements and vice versa. As such, this problem will be one of my principal concerns, requiring a logical analysis of the relevant aspects of the propositions involved. But before attempting to make a statement of my position on these matters, the contemporary origins of the dispute will be traced in an examination of McTaggart's argument; or rather, of Dummett's exposition of the argument,[2] since his interpretation predominates in what follows.

2. McTaggart's Argument

McTaggart suggests that there are two ways in which the positions of events in time can be expressed. On the one hand, an event may be past, present or future, and on the other, it may be earlier or later than some other event. The term 'A-series' is introduced for "that series of positions which runs from the far past through the near past to the present, and then from the present through the near future to the far future", and the term 'B-series' for "the series of positions which runs from earlier to later".[3] Propositions of the kind 'M is past (present, future)' are called A-propositions or A-statements, and propositions of the kind 'M precedes (is earlier than) N' are called B-propositions or B-statements.

Dummett observes that the essence, if not the elegance, of McTaggart's argument does not depend upon being stated in terms of events. Although there may be problems in characterising the notion of an event and describing general procedures for transforming propositions making reference to events into ones that do not, the argument can be restated in terms of tensed propositions in which the predicates 'past', 'present' and 'future' do not appear. Thus, 'The paper's being white is past' becomes 'The paper was white'; and in general 'past', 'present' and 'future' can be replaced by 'was', 'is' and 'will be' together with appropriate grammatical modifications. For this reason, tensed statements are also included amongst A-propositions.

Similarly, B statements containing expressions which refer to events can be replaced by ones which do not. For instance, 'The birth of Queen Anne is earlier than her death' becomes 'Queen Anne is born at t_1 and dies at t_2, and t_1 is earlier than t_2', or simply 'Queen Anne is born before she dies'. And just as tensed statements were included amongst A propositions, so tenseless statements of the

[2] Michael Dummett, "A Defense of McTaggart's Proof of the Unreality of Time", *Philosophical Review*, 69 (1960).

[3] J.M.E. McTaggart, *The Nature of Existence*, Vol. II (Cambridge, 1927) Book V, chapter 33. Reprinted in Gale (ed.) *The Philosophy of Time*, (London, 1968). Pp. 87–88.

kind just mentioned, together with tenseless dated statements of the kind 'Queen Anne dies at *t*', are counted as B-statements.

With this terminology, McTaggart's argument proceeds in two parts. First, he argues that A-propositions are essential for a description of time by arguing that they cannot be reduced to B-statements, whilst if there were no facts of the kind expressed by A-propositions, there would be no time at all. For time essentially involves change; but only A-statements can express change. B-statements express facts which are permanent and therefore cannot express change: "If M is earlier than N, it is always earlier".[4] But an event does change in one respect: "It was once an event in the far future. It becomes every moment an event in the nearer future. At last it was present. Then it became past, and will always remain past, though every moment it becomes further past".[5]

"Having, as it seems to me, succeeded in proving that there can be no time without an A-series", McTaggart continues, "it remains to prove that an A-series cannot exist, and therefore that time cannot exist".[6] This is to be established in the second part of his argument by showing that A-statements involve a contradiction. The predicates 'past', 'present' and 'future' are, so the argument runs, on the one hand incompatible predicates, and on the other, all three applicable to every event. It will not do, McTaggart argues, to object that although all three predicates apply to each event, they do so at different times, by saying that an event *is* past, but *has been* future and present; or that it *is* future, but *will be* present and past. For this means that the predicates which apply are not the simple 'past', 'present' and 'future' but, for example, 'will be past', 'is present' and 'was future'. However, the antagonist's argument advances us no further, since "instead of three, we now have nine predicates, each of which still applies to every event and some of which are incompatible, for example, the predicates 'was past' and 'will be future'."[7] Any further objection that the predicates which really apply are 'is going to have been past' and 'was going to be future' which are compatible can be countered as before, and so on indefinitely.

3. Critique of the First Part of McTaggart's Argument

The argument rests on the distinction between A- and B-statements, or between the A- and B-series. Following Dummett, I take it that the significant difference between the two classes of statements lies in the fact that tenses, in common with expressions such as 'I', 'you', 'this', 'now', etc., endow sentences in which they occur with the characteristic that the truth values of the statements made by the use of such sentences depend upon the circumstances in which they are used. Ex-

[4] McTaggart, loc. cit.
[5] McTaggart, op. cit., pp. 90–91.
[6] McTaggart, op. cit., p. 94.
[7] Dummett, op. cit., p. 497.

pressions with this property are called indexical expressions, or simply indexicals. Having said this, one of the claims made in the first part of the argument, that A-statements are not reducible to B-statements can, I think, be shown without resorting to McTaggart's appeal to the notion of change.

According to Russell's account of A-propositions, temporal indexicals are a species of what he calls 'emphatic particulars'[8] or 'egocentric particulars'[9] and can be reduced to a single indexical 'this'. Roughly, 'this' is taken to mean 'what is present to me' or 'what I am now noticing', and the account is developed variously in the references cited. An assertion that an event is present means, on this view, that the event is simultaneous with *this,* whilst to assert that it is past or future is to assert that the event is earlier or later than *this*. Reichenbach's so-called 'token-reflexive' analysis of tenses[1] is a more sophisticated version of this theory. According to his theory, a present tensed statement says that an event occurs now, and 'now' is defined as 'the time at which this token is uttered', where 'this token' refers to the sentence token in which this expression appears. Past and future tensed statements are interpreted as saying that an event occurs earlier or later than now, although, as we shall see later, the analysis is somewhat more complicated than the exposition so far might lead us to expect. Adapting an argument of Castañeda's,[2] this analysis of the indexical expression 'now' can be succinctly refuted. For according to normal usage, the statement 'The utterance of this token does not occur now' is false, but contingently so (it is possible that the utterance of this token does not occur now, although in actually uttering this token it is falsified), whereas the statement formed by replacing 'now' with its proposed definiens, 'The utterance of this token does not occur at the time of utterance of this token' is self-contradictory—it cannot possibly be true. The point is that the truth of an A-statement—to the effect that an event occurs now, for instance—does not carry the implication that some utterance actually occurs, whereas the proposed definiens does. Furthermore, it can be quite generally argued that A-statements cannot be reduced to B-statements, which do not involve indexical expressions, since logical connectives and quantifiers cannot operate on expressions containing no indexicals to produce statements which do involve such expressions, since logical connectives and quantifiers are not themselves indexical expressions.

It is important that this claim of the first part of the argument be established along these lines, since McTaggart's own argument by appeal to the notion of change is completely ineffective against B-theorists who maintain that B-propositions are adequate to express all facts. McTaggart assumes that change

[8] Bertrand Russell, "The Philosophy of Logical Atomism", *Monist*, 28–29 (1918–1919), lecture IV, and "On the Experience of Time", *Monist*, 25 (1915).

[9] Bertrand Russell, *An Enquiry into Meaning and Truth*, (London, 1940), chapter 7.

[1] Hans Reichenbach, *Elements of Symbolic Logic* (New York, 1947), pp. 284–298.

[2] Hector Neri Castañeda, "Indicators and Quasi Indicators", *American Philosophical Quarterly*, 4 (1967).

can be expressed only by propositions which may change their truth values from time to time:

> If, as I have maintained, there can be no change unless facts change, then there can be no change without an A-series... There can be no change unless some propositions are sometimes true and sometimes false. This is the case of propositions which deal with the place of anything in the A-series—'the battle of Waterloo is in the past', 'it is now raining'. But it is not the case with any other propositions.[3]

B-propositions do not change their truth values from time to time—according to McTaggart, because "If M is earlier than N, it is always earlier"[4]—and consequently they cannot express change in McTaggart's sense. But a B-theorist would reject this account of change. He can express change by saying, for instance, that some individual is φ at some time t and at some time t', later than t, this individual is not φ,[5] and McTaggart has not advanced any arguments which would lead him to reject this account of change.

Some philosophers have wanted to maintain the conclusion of the first part of McTaggart's argument, that A-propositions are essential, by arguing that not only are A-propositions not reducible to B-propositions; B-propositions are in fact reducible to A-propositions. Clearly some such additional argument is necessary since although A-propositions are not reducible to B-propositions, it might well be that B-propositions are not reducible to A-propositions either, in which case the indispensability of A-propositions will not have been established. In order to eliminate tenseless statements, Wilfred Sellars has proposed that the tenseless 'is' can be understood in accordance with the schema:

x is φ at t. ≡ . Either x *was* φ at t or x *is* φ at t or x *will be* φ at t.[6]

However, I fail to see how the right hand side of this equivalence throws any light on the meaning of the tensed copula. For the first disjunct 'x was φ at t' must, presumably, mean 'x is (tenseless) φ at t *and* t is in the past'. Otherwise, the only alternative seems to be that the time of x's φ-ing at t is in the past—for example, 'It was the case that Bertrand Russell dies on the 2nd February 1970'. But this gives two time indications where only one is required, namely the time at which x φ's—there is no time at which x's φ-ing at time t occurs; no time at which Bertrand Russell's dying on 2nd February 1970 occurs. Accordingly, since the other disjuncts must be interpreted similarly, the right hand side of the equivalence is equivalent to 'x φ's at t and either t is past, present or future'. Now, the second conjunct here is tautologous, since every moment is either in the past, the present

[3] McTaggart, op. cit., p. 93.
[4] McTaggart, op. cit., p. 87.
[5] See, for instance, J.J.C. Smart, "The River of Time", *Mind,* 58, (1949).
[6] Wilfred Sellars, "Time and World Order", *Minnesota Studies in the Philosophy of Science,* vol. III. (Minneapolis, 1962), p. 533.

or the future. But an equivalence like '$q \equiv (q \,\&\, (p \vee \sim p))$' does not tell us much about q.

Objections to some of our B-statements, that they are not formulated in the most natural English, can be countered in the same way. It is more natural, for instance, to say 'Queen Anne *died* in 1714' than to say 'Queen Anne dies in 1714'. But this is quite innocuous; it merely reflects the fact that we usually want to say both that Queen Anne dies in 1714 and that she died at some time in the past. This may be taken to suggest that the classes of A- and B-statements are not mutually exclusive amongst all propositions making reference to positions in time. But it also suggests that propositions having both A- and B-characteristics can be seen to comprise what might be called atomic statements having entirely A-characteristics or entirely B-characteristics. This matter will be considered later.

At this point it is possible to see more clearly why McTaggart's argument by appeal to the notion of change is inadequate. His mistake lies in the inference from 'M is earlier than N' to 'M is always earlier than N'. For like x's φ-ing at t, M's being earlier than N is not the sort of thing which holds at a time, and there is no moment at which M is earlier than N. This implies, of course, that given any moment t, it is not the case that 'M is earlier than N' holds true at t; but then the latter does not imply that 'M is earlier than N' is false at t. B-statements do, for this reason, express permanent states of affairs in McTaggart's sense, since there is no possibility that B-statements have different truth values at different times. But it is gratuitous of McTaggart to assume that all propositions making reference to positions in time are propositions of which it makes sense to say that they hold true at a time.

It appears that McTaggart was himself one of those who thought that B-statements could be reduced to A-statements, and therefore that the first part of his argument could be established independently of his appeal to change. Prior reports[7] that later on in *The Nature of Existence*, after his proof of the unreality of time, McTaggart suggests that M is earlier than N if ever M is past whilst N is present, or present whilst N is future. This suggestion has been adopted by a number of philosophers who propose that B-statements are reducible to some such disjunction of tensed propositions, although more elaborate than McTaggart's.

It seems to me, however, that reductions of this kind are inadequate, and that in general tenseless statements cannot be reduced to statements making reference to positions in time entirely by indexicals. If statements containing the 'earlier than' relation are to be counted as B statements, and statements containing temporal indexicals are to be counted as A statements, as the use of this terminology so far suggests, then it seems to me that not every statement making reference to positions in time falls into just one of these two categories. Moreover, not every such statement can be seen to comprise a conjunction or disjunction of propositions which are either A or B statements but not both, as was suggested

[7] Prior, op. cit., p. 4.

above—statements in which the verb occurs in the future perfect or the imperfect are cases in point. My reasons for holding this view will become apparent in later chapters when I have presented my analysis of tensed statements. However, I shall criticise shortly an argument that has been proposed in favour of the thesis that B-statements are reducible to A-statements. If I am right about this, we should become rather sceptical of the rigid distinction drawn between A- and B-statements before the reductions to include one class of statements entirely within the other are attempted.

4. Critique of the Second Part of McTaggart's Argument

Turning now to the second part of the argument, the claim here is that 'past', 'present' and 'future' are incompatible, yet all three apply to each event. The most natural objection is to dispute the second part of this claim, that all three predicates apply to every event. McTaggart did, as we have seen, consider an objection along these lines, to the effect that all three predicates apply to any event at different times. But this only resulted in an infinite regress which defeats the objection. Nevertheless, the dispute can be pressed in a way not susceptible to the infinite regress simply by maintaining that any event is either past, present or future, where the logical connective is the exclusive 'or'. The claim is now no longer that all three predicates apply to the same event, but at different times. If McTaggart were now to introduce a second tense inflexion to form more complex predicates, some of which he claims are incompatible, such as 'was past' and 'will be future', the antagonist can reply that any event was past or will be future, but that this involves no contradicition since the 'or' is again exclusive. Any further suggestion that another tense inflexion should be introduced, resulting in even more complex but incompatible predicates, can be countered as before, and so on indefinitely.

Dummett discusses another objection to McTaggart's argument which, he concludes, is just a reformulation of the original objection McTaggart considered and which can be refuted on the same grounds. This reformulation is not, however, the reconstruction just considered and Dummett's qualms about the second objection are not applicable to that reconstruction. For two predicates containing indexical expressions, so this second objection runs, are incompatible if it is contradictory to predicate both of them of the same object under the same circumstances. But although 'past', 'present' and 'future' are incompatible in this sense, no contradiction arises so long as they are not predicated of the same event under the same circumstances. Thus, to take an example of Dummett's, the paper was white and is yellow, and the predicates 'white' and 'yellow' are incompatible in the sense just mentioned; but no contradiction arises because 'white' and 'yellow' are predicated of the paper under different circumstances. However, Dummett observes that the original objector held that a contradiction arising at any level of the hierarchy of predicates constructed in the infinite regress could be resolved by ascending to the

next level. According to the present objection, the original objection can be seen as an attempt to specify the circumstances in which the predicate is asserted of the event. McTaggart's reply to the original objection can now be reformulated as the claim that any such attempt fails because the specification of the circumstances was itself by means of indexical expressions, and all that has been achieved is the construction of new predicates of the same type. This reformulation does not counter the reconstruction developed in the previous paragraph since the antagonist need not specify at what time or under what circumstances the incompatible predicates apply to any event. He need only say of any event that it is past, present or future, with the exclusive sense of 'or'. The onus is now on McTaggart to say that under such and such circumstances the event is past, under other circumstances it is present and under still different circumstances it is future; but as before, this does not advance his argument. The point of the present objection is perhaps better expressed by saying of an event that, under the present circumstances, it is either past, present or future, again with exclusive 'or'. But since this does not amount to saying that under past circumstances the event is present, and under present and future circumstances it is past, etc., the contradiction is not being denied by ascending the first step of an infinite regress, and so the reformulation of McTaggart's reply carries no weight against this objection.

The objection presented here to the second part of McTaggart's argument is related to the discussion in the previous section of his appeal to the notion of change in the first part of the argument. Furthermore, it brings to light a fundamental criticism of orthodox tense logic which encompasses an analysis akin to McTaggart's views on tenses. The locus of our criticism of the first part of the argument revolved around McTaggart's insistence that change must be expressed by propositions which may change their truth values from time to time. This notion of a proposition with variable truth value is adopted approvingly by Prior as a basic tenet of his account of tenses:

> In a rationalised language with uniform constructions for similar functions we could form the past tense by prefixing to a given sentence the phrase 'It was the case that'. . . and the future tense by prefixing 'It will be the case that'. For example, instead of 'I will be eating my breakfast' we could say 'It will be the case that I am eating my breakfast', and instead of 'I was eating my breakfast' we could say 'It was the case that I am eating my breakfast'.[8]

Prior makes it clear that 'I am eating my breakfast', as it occurs in a sentence of the kind 'It was the case that I am eating my breakfast', is to be represented by a propositional variable which may have different truth values at different times[9] and is to be understood in the present tense:

> The construction I am sketching embodies the truth behind Augustine's suggestion of the 'secret place' where past and future times 'are', and his insistence that

[8] Prior, "Changes in Events and Changes in Things", reprinted as chapter 1 of *Papers on Time and Tense*, (Oxford, 1968), p. 8.

[9] Prior, *Time and Modality*, (Oxford, 1957), p. 8, and *Past, Present and Future*, pp. 14–16.

> wherever they are, they are not there as past or future, but as present. The past is not the present but it *is* the past present, and the future is not the present but it *is* the future present.[1]

Already the resemblance between Prior's analysis and the construction McTaggart thought his opponent to the second part of his argument must endorse is becoming apparent. For in order to dispel the contradiction between 'past', 'present' and 'future' the objector claims that the predicates which really apply are, for example, 'will be past' and 'was future'. These predicates are rendered by McTaggart as 'past in the future' and 'future in the past', and so on. Dummett represents the hierarchy of predicates which this procedure generates by the schema:

$$\left\{\begin{matrix}\text{past}\\\text{present}\\\text{future}\end{matrix}\right\} \text{ in the } \left\{\begin{matrix}\text{past}\\\text{present}\\\text{future}\end{matrix}\right\} \text{ in the } \left\{\begin{matrix}\text{past}\\\text{present}\\\text{future}\end{matrix}\right\} \text{ in the...}$$

That Prior whole-heartedly endorses this hierarchical structure is clear since he continues:

> There is also, of course, the past future and the future past. For these adverbial phrases... can be applied repeatedly... Hence, we can have such a construction as 'It will be the case that (it has been the case that (I am taking off my coat))', or in plain English, 'I will have taken off my coat'.[2]

The point of my objection to the second part of the argument was precisely to avoid such notions as 'past presentness', 'past futurity', and so on. Only failure to grasp this point would lead anyone to criticise this move along the lines of the putative reply of McTaggart's mentioned earlier by ascending the first level of the hierarchy. I have already argued that tenseless statements of the kind 'M is earlier than N' and 'M occurs at time *t*' are not, *contra* McTaggart, statements which are true or false at a time. Similarly, it seems to me that tensed statements do not hold true or false at a time in the sense that one and the same tensed statement may vary in truth value from time to time. This is not to say, of course, that one and the same sentence cannot be used on different occasions to make statements which have different truth values. For this is not the same as allowing statements to vary their truth values with time since different statements are made on different occasions by the use of one and the same sentence. To say that an event is present or future is not, on this view, to say that the event is future in the past; neither does an event's being present or past imply that it is past in the future. 'Past', 'present' and 'future' make reference to positions in time, on this view, with respect to the time when the proposition in which they occur is made, and this point of reference dominates all subordinate clauses within the proposition. Prior says of his own views just mentioned that English speakers find it hard to see these things clearly:

[1] Prior, "Changes in Events and Changes in Things", p. 8.
[2] Ibid.

> . . .for in English sentences the point of view of the speaker dominates even subordinate clauses. When an English speaker, for example, wants to say on Tuesday that someone complained on Monday of a sickness that he had that day, the correct form of words will be 'He said he *was* sick', although the man was in fact complaining not of a then-past but of a then-present sickness, and his own words would have been 'I am sick'.[3]

On the view I am proposing, there is no need to castigate English in the way Prior suggests, paraphrasing the sentence in question as 'He said that he *is* sick'. For there is no danger that 'He said he was sick' be interpreted in any way which would suggest that his sickness is in the past past.

Prior does, of course, make the substantial claim that his views lead to a systematisation of our use of tenses, which has been barely hinted at in the foregoing passages. The most plausible example we have come across so far is his analysis of the future perfect sentence 'I will have taken off my coat'. But I will be able to show in the ensuing chapters that it is possible to develop the views presented here in such a way that the more complex tenses such as the future perfect can be analysed without resorting to a hierarchy of tensed predicates of the sort McTaggart envisaged.

5. On the Relation between Tensed and Tenseless Statements

Some such objection to the second part of the argument of the sort considered by Dummett rests, he maintains, on a grave misunderstanding of McTaggart. "If it gave a correct account of the matter, then only stupidity could explain McTaggart's failure to use a quite analogous argument to show the unreality of space and the unreality of personality."[4] 'Here' and 'there', like 'near' and 'far', are incompatible in the sense explained, but every place is both 'here' and 'there', both 'near' and 'far'. Yet McTaggart displays not the slightest inclination to apply his argument to space. Dummett suggests that the analogue of the first part of the argument, to demonstrate the indispensability of temporal indexicals, does not hold for space; and since the second part depends upon the first part, the spatial analogue of the second part has no force. The conclusion Dummett draws from this is that McTaggart took it for granted that reality is that realm of things of which there exists a complete description – by which is meant an observer-independent description. Time, however, cannot be adequately described, so the first part of the argument is intended to establish, without the use of indexical expressions; consequently, if time were real, there would not be a complete, observer independent description of reality after all. Space is held to be real because it is possible to give a complete description of spatial relations and positions without reference to the observer, whereas indexicals play an essential

[3] *Past, Present and Future*, p. 14.
[4] Dummett, op. cit., pp. 499–500.

role in any language purporting to give a complete description of time and so time is held to be unreal.

On this interpretation, then, the first part of the argument is crucial to McTaggart's thesis. But we have seen that the argument to establish the indispensability of A-statements on the grounds that it is impossible to express change without them was unconvincing, and I have suggested that the thesis cannot be established by showing that tenseless statements are reducible to tensed statements but not vice versa. Apart from the discussion of Sellars' proposal, I have said that my reasons for making this claim will not become fully apparent until my analysis of tenses is developed later. However, something may be added to the argument at this stage by way of disputing an argument that has been advanced in favour of the conclusion to the first part of McTaggart's argument.

Richard Gale[5] has argued in a similar vein to the view Dummett attributes to McTaggart, that an observer-independent description of time is incomplete. Gale argues that A-propositions can serve to convey certain factual information, which would be lost if the corresponding B-propositions were used, by illustrating the informative role A-statements play in situations immediately relevant to the coordination of actions to achieve some shared goal. Thus, Joe—a scout for a machine-gun company—alerts the company commander with the A-statement 'The enemy is now within a hundred yards'. The same information cannot, according to Gale, be conveyed by the corresponding B-statement 'The enemy advances (tenseless) within a hundred yeards at time *t*', since the company commander must check his watch in order to discover whether the time *t* is past, present or future. Accordingly, the tenseless dated statement does not convey certain information about the event in question, namely that it is occurring now. Nor does this statement entail a statement which does serve to convey such information. Although the tenseless dated statement does convey information which makes it possible for the listener to discover, by nonlogical means, whether the event in question has occurred, is occurring or will occur, this information, according to Gale, is not part of the information actually conveyed by the statement. Consequently, statements of the kind 'M is happening now' and 'M happens (tenseless) at *t*' differ in informative content, even if they are both made at the same time *t*, and Gale concludes that A-statements are not reducible to B-statements.

Gale goes on to propose that B-statements are reducible to a disjunction of A-statements similar to that of McTaggart's already mentioned. This particular claim will be examined later; for the present I want just to consider what follows from Gale's argument about the relation between A- and B-statements.

Perhaps those who propose a reduction of B-statements to a disjunction of A-statements have in mind the fact that disjunction has the effect of suppressing information. This would account for the additional information conveyed by an A-

[5] *The Language of Time* (London, 1968), chapter IV.

statement not being conveyed by the corresponding B-statement. But if this is why the definition has been proposed, the argument about the relative factual content of A- and B-statements does not support it. For Gale's argument establishes only that 'M is happening now' and 'M occurs (tenseless) at time *t*' are not logically equivalent—that it is possible for one to be true and the other false. It has not been established that either statement entails the other, and consequently it does not follow that one statement is more informative than the other. All that follows is that the two statements may serve to convey different information. Nevertheless the two statements are not entirely unrelated since it is clear from the situation Gale describes that there are circumstances in which they are equivalent: if it is now *t*, then 'M is happening now' is equivalent to 'M occurs (tenseless) at *t*'. Under this condition, neither the tensed nor the tenseless statement is more informative than the other since they are equivalent.

A similar situation obtains with respect to indexical and non-indexical expressions referring to positions in space: if place *x* is here, then 'M occurs here' is equivalent to 'M occurs at place *x*'. But this is not sufficient to show that, as has sometimes been claimed, statements containing spatial indexicals are reducible to statements containing no such expressions. Perhaps now we should carefully distinguish the question whether temporal indexical expressions are dispensable for a complete description of time from the question of the reducibility of indexical to non-indexical descriptions of time, and vice versa. Can one of the classes of A-statements and B-statements be reduced to the other? The considerations of this section suggest that this is not possible. It follows that A-statements are not eliminable since B-statements do not have the same sense. But does it also follow that a description of time would be incomplete without the use of indexical expressions? Do temporal indexicals add any more to a description of the structure of time than could be achieved entirely by the use of tenseless statements?

In order to answer these questions something has to be said about the criteria for completeness of a description of time. But in view of the point of analogy we have found between expressions referring to spatial and to temporal positions, the criterion Dummett outlined—that a complete description must be observer-independent—appears now not so clear. For his notion of completeness depended upon taking space as a paradigm of something for which there is an observer-independent description, and time is simply said to be disanalogous to space in this respect. But the possibility of an observer-independent description of space was maintained on the grounds that spatial indexicals are eliminable in favour of non indexical expressions, and, as we have seen, this view cannot be sustained since the nearest we can get to a definition is only a conditional equivalence of the kind mentioned above. Moreover, on the point of the eliminability of indexicals, it is maintained here that space and time are in fact analogous in this respect: all we have is a conditional equivalence between indexical and non-indexical expressions, which is not strong enough to sanction substitution in all contexts *salva veritate*. So far as this notion of completeness goes, depending as it does on index-

ical expressions, it would appear that time is just like space. Consequently, since the disanalogy cannot be maintained at the crucial point, we cannot appeal to space to make good this sense of completeness which is said not to apply to time.

This does not as yet give us an answer to the question whether a description of time would be incomplete without the use of indexicals; it only enables us to dispute the affirmative answer Dummett attributes to McTaggart. However, I think it might be possible to maintain that there is a sense in which a description of time could be complete without using indexicals. Consider again Castañeda's argument to show that 'now' cannot be eliminated in favour of the expression 'the time at which this token is uttered'. We said that whereas

(1) The utterance of this token does not occur now

is contingently false, the statement obtained by substitution according to the proposed definition

(2) The utterance of this token does not occur at the time of utterance of this token,

is necessarily false since it is self-contradictory, and so we have some modal contexts where substitution is not possible *salva veritate.* But this argument must be examined more closely, for it seems to me that although there is a sense in which (2) is necessarily false, this is to construe the modality in its *de dicto* sense. If we take the modality in the *de re* sense, (2) is possible just as it is possible that the horse that was disqualified in the Kentucky Derby lacks the property of being disqualified in the Kentucky Derby. For being disqualified in the Kentucky Derby is hardly an essential property of the horse that, as a matter of fact, was disqualified from the Kentucky Derby.[6] Similarly, 'occurring at the time of utterance of this token' is probably not an essential property of this token; nor, for that matter, is 'being the time at which this token is uttered' an essential property of the time which is, in fact, the time when the token in question is uttered. So (2) is possible, though false; and (1) is possible, though false, in the same *de re* sense since it is not an essential property of the token in question that it occurs now. Understood in the *de re* sense, then, (1) remains contingently false under the definitional substitution giving (2), and consequently these are not contexts, under the suggested interpretation, where substitution *salva veritate* fails.

It would appear, then, that the expressions 'now' and 'the time at which this token is uttered' designate the same thing, though the way in which a language user understands what these expressions designate may well be different, which we might express by saying that they have different senses. Thus, the truth of a statement to the effect that an event occurs now, for example, does not imply that some utterance actually occurs. Rankin expresses this point as follows:

[6] Richard Cartwright uses this example in "Some Remarks on Essentialism", *Journal of Philosophy,* 65 (1968).

> The most that a speaker A does for his token in stating 'Brutus killed Caesar' is to offer that token to his listener B for subject matter so that B can say 'Brutus killed Caesar before A made his statement'. Here, however, B's subject matter is in excess of A's. If on the other hand B had wished merely to corroborate A's statement all he would have needed to do would have been simply to repeat it. Both A's and B's statement would then have exactly the same subject matter. This is because both A and B offer their own tokens and accept the other's for purposes of identification without referring. . . to either.[7]

The detailed arguments Rankin gives in this article against the characterisation of indexical expressions as token-reflexive can be taken, I think, to bring out a difference in sense between the expressions 'now' and 'the time of utterance of this token' (and how 'this' is to be understood in this phrase) which becomes apparent when we consider the presence of the speaker and the listener as well as the subject matter communicated. (It is interesting to note that Rankin, in accordance with the position adopted here, maintains that temporal indexicals are no different from spatial indexicals in this respect.)

Returning to the question of the completeness of time, I cannot see that any consequences for the completeness of a description of time follow from the fact that tenses are indexical expressions for which there are no equivalent expressions not involving temporal indexicals which can replace them in any communication situation. Time is like space in this respect. Any difference between a description of time which employs indexical expressions and one which does not would seem to amount to a difference in sense, so that we might perhaps understand how the descriptions refer to time in different ways. What is designated by the descriptions, however, is the same, and so in this sense we can say that a description of time is not incomplete simply in virtue of not using indexical expressions, for there is nothing that can be designated by temporal indexicals which cannot also be designated by non indexical expressions (this is what the conditional equivalence shows).

In the sequel I shall develop a logic of temporal reference designed to express both tensed and tenseless statements in which this conditional equivalence can be expressed as a valid formula. Within this system we shall see the rigid distinction between A- and B-statements, and between the A- and B-series, collapse.

[7] K. W. Rankin, 'Referential Identifiers', *American Philosophical Quarterly*, *1* (1964), pp. 240–241.

Chapter II

Tenses in English

1. Preamble

We drew a distinction in Chapter I between tensed and tenseless statements. The distinction was characterised there by taking tensed statements to refer to positions in time by making essential use of indexical expressions, whereas tenseless statements are taken to make no such use of these expressions (ignoring personal indexicals, spatial indexicals, and so on). My task in this and succeeding chapters is to try to make this distinction more clear and precise. In the present chapter I shall consider the expression of tenses in English, and try to locate the indexical character of tensed statements more precisely, by way of motivating the formal system presented in the following chapter.

It is clear that the notion of tense at issue in the preceding chapter is not that of a grammarian concerned to classify all the verb forms, together with their auxiliaries, into a small number of categories; for example, the traditional categories of the preterite, the pluperfect and the future perfect. For sentences used to express what were there called tenseless statements—of the kind '*x* is B at *t*', for example—clearly do have a grammatical present tense, and are in fact regarded as present tensed by grammarians, who sometimes take such statements as limiting cases of the use of the specious present. Indeed, it is sometimes said that there are just two tenses in English: the past and the non-past. There is no future tense in English, according to this view, since apparently future tensed sentences are compounded of verbs with present tense (non-past) inflexions. In contradistinction to views of this kind, the notion of tense at issue here is concerned with what certain word forms are used to express, and there certainly are tenseless statements and future tensed statements expressed by the use of English sentences, even if the linguistic means available for expressing them is perhaps not so clearly marked as in other cases.

Moreover, it would be extremely misleading from the point of view of the present concern to follow too closely linguistic forms and classify the tense of a particular verb in use as, say, a past tense simply because it has the grammatical form 'have -ed', as in 'He has finished his supper'. For this particular verb inflexion need not necessarily be used to refer to a moment of past time; for example, when embedded in a larger context, *viz.* 'When he has finished his supper, he will go to his room'. Similarly, the grammatical present tense can be used to refer to a moment of future time in the sentence 'The ship docks at midnight'; and there is the

so-called 'historic present' where the present tense is used to refer to occasions in the past. As a final example in this vein, the sentence 'John would arrive at five' might be used to express an often repeated or habitual practice of John's; but not so in the larger context 'John said that he would arrive at five' where John's arriving is said to occur some time after he spoke. These examples are sufficient to illustrate that the meaning or tense expressed by the use of a grammatical tense does not depend simply upon the form of the words used, and so there is a distinction of some sort to be drawn here.

These precautions having been noted, however, the two notions of tense are not entirely independent, and it must be admitted that the variety of grammatical tense forms does indicate a structural complexity in what is expressed by the use of such tenses to which a three-fold distinction between past, present and future hardly does justice. Nowadays at least, some linguists have wanted to draw a distinction between tenses conceived as linguistic structures and what such structures may be used to express, which is sometimes characterised in the wake of Chomsky's theories as a distinction between "deep" and "surface" tenses.[1] Accordingly, we might well expect that the work of certain linguists should throw some light on why, for example, we need all the three forms 'played', 'have played' and 'had played'—what sort of distinction of sense can be expressed by these three forms such that just one form of the past tense will not do? Prior, as we have seen, tackles differences of this kind by allowing iterations of his simple forms in order to generate constructions such as the 'past past', the 'past future', and so on. But this sort of construction violates the principle enunciated in Chapter I that the point of view of the speaker dominates all subordinate contexts. Now William Diver, in a linguistic study of tenses in English,[2] has produced an account in which I think we can see the bones of this principle at work, and I shall go on to outline and comment on the main thread of his very suggestive paper. Following this I shall go on to discuss Reichenbach's analysis of tenses and present a modified version of his theory in the light of some of Diver's suggestions which will eventually come to fruition in the third chapter in the form of a logic of tenses.

2. Diver's Analysis of Tenses in English

After considering and rejecting the analyses of a number of other linguists, on the grounds that "the rules and definitions given do not describe the actual usage of the language", Diver proposes his own account of tenses for which he makes the bold claim of offering "rules that are without exception".[3] He sets about this task

[1] See, for example, Sven Jacobsson, 'Five Categories of Tense' in his *Studies in English Transformational Grammar* (Stockholm 1971), and Rodney Huddleston, 'Some Observations on Tense and Deixis in English', *Language*, 45 (1969).

[2] William Diver, 'The Chronological System of the English Verb', *Word*, 19 (1963).

[3] Ibid., p. 152. All further references to this article in the present section are indicated in the body of the text.

by correlating the linguistic forms of verbs (in the active mood only) with the meanings these forms are used to express, in so far as these linguistic forms are used to express reference to and relations between moments in time. The results are set out in a table, an illustrative fragment of which is set out below in a slightly different format. We have just seen that meaning is not entirely a function of word form since the same form may be used on different occasions and in different context to express more than one meaning, so it comes as no surprise to find that word forms sometimes appear in more than one place in the table.

An Illustrative Fragment of Diver's Table

SEQUENTIAL TENSES	AFTER, AFTER	would be -ing	will be -ing	Ø
	AFTER, SIMULTANEOUS	would be -ing	will be -ing	Ø
	AFTER, BEFORE	would have -ed	will have -ed	Ø
	AFTER	would -O, was -ing + non-simultaneous	will -O, is -ing	will be -ing
	SIMULTANEOUS	was -ing	is -ing	will be -ing
	BEFORE	had -ed -ed	have -ed -O	will have -ed will -O
		PAST	PRESENT	FUTURE
NON-SEQUENTIAL TENSES		-ed	Ø	-O, is -ing, will -O, will be -ing
	INDEFINITE	have -ed	-O	Ø
	DEFINITE	was -ing	is -ing	Ø

A circle represents the uninflected form; for example, the 'O' form of 'to walk' is 'walk', and so on. The forms given are all in the first person singular, active mood and will require appropriate modification with different subjects, moods and where the verb is irregular.

It seems that Diver's notion of a tense is that of a linguistic form of words—namely, the inflexion of a verb together with its auxiliaries, if any. The analysis is then semantical, so that tenses understood in this sense are correlated with the meanings they express. Now it would seem that the latter notion—i.e. the meaning expressed by tenses in Diver's sense—is more akin to McTaggart's notion of a tense than is a linguistic construction, and is more like the notion of a tense to be developed in this essay. Accordingly, our prime interest in Diver's work will be in the semantical structure he develops rather than the details of the correlation between linguistic units and the meanings they can be used to express. However,

we shall make some comments on the delimitation of the class of linguistic units that might appropriately be considered tenses in Diver's sense.

Diver's original table is set out in three columns in such a way that just one of the three elements of meaning 'past', 'present' and 'future' is common to each member of a column, and so that the members of any horizontal row are in some sense analogous to one another, having some elements of meaning in common. So we find, for example, the three forms 'had -ed', 'have -ed' and 'will have -ed' in a row, corresponding in this case to a traditional distinction amongst perfect tenses, and having in common the characteristic of expressing that some event occurs before some other time, but differing in that this 'other time' is in, respectively, the past, the present and the future. Note that the terms 'past', 'present' and 'future' as used here do not necessarily apply to the time at which the event described by the verb occurs. Rather, a linguistic tense is said to be used to express some temporal relation to a moment in the past, the present or the future.

Taking out the common elements of the rows and columns of Diver's table and placing them at points on two perpendicular axes enables us conveniently to represent his scheme by a two-dimensional graph or matrix, where the verb forms occur at the appropriate points of intersection of the elements of meaning they are used to express. Some gaps appear (marked here by 'Ø') in the present column and even more in the future column. Now Diver's general approach is to take the past column as a prototype and to consider the degree of analogy between members of this column and members of the present and future columns. Accordingly, the distribution of gaps in the table is taken by Diver as an indication of a greater lack of analogy between future and past tenses than between present and past tenses. As an illustration, Diver holds that there is no verb form (i.e., no tense in his sense) which expresses an analogue in the future column of what is expressed in the past column by 'would have -ed' and in the present column by 'will have -ed'. That is to say, there is no verb form which expresses that an event occurs before some time after another time in the future.

Members of any horizontal row are related by "oppositions of exclusion", which Diver explains as follows:

> No combination of the meanings 'past', 'present' and 'future' can occur, either as the complex meaning of a signal in the verbal system or by combination of a signal with some extra verbal element; that is, the meaning of 'past' is incompatible with the meaning of 'present' and 'future' in reference to the same event. Hence, within the verb, there is no such signal as '*He will walked' with the meaning 'past, future'. Extra verbally such sentences as '*He walked tomorrow' do not occur. (p. 153)

This looks very much like our principle from Chapter I that the point of view of the speaker dominates all subordinate contexts, thereby excluding constructions such as the 'future past' and so on. For, in view of the foregoing, Diver will clearly not be able to avail himself of iterated sequences of the element 'past', 'present' and 'future' in the analysis of traditional tenses such as the past perfect and the future perfect. Instead, he resorts to further distinctions of sense—represented in

our matrix by the elements along the vertical axis—in order to provide an analysis of the more complex tense constructions. Some of these finer points of meaning to which Diver draws attention will prove useful in the analysis presented in the sequel, though, it should be emphasised, more for their suggestiveness than the degree of precision with which they are expressed.

Before going on to comment on some of the details of Diver's analysis, however, I would like to indicate some reservation on the point of the incompatibility of the elements 'past', 'present' and 'future'. It seems that Diver construes this incompatibility as demanding that all uses of linguistic tenses be classified in just one of the categories of past, present and future, and then he further distinguishes different uses within this three-fold division. Now, I don't think that in general this ternary structure will do. For problems arise for the logician, if not for the linguist, in ascribing tenses to some relational predicates. What, for example, is the tense of the two-place predicate 'is taller than' in the sentence 'I am taller than my grandfather was'? As we shall see, I think it is possible to maintain the incompatibility of 'past', 'present' and 'future', avoiding iterations of these terms, but that this does not require that every tensed predicate be regarded as past, present or future tensed, where the 'or' here is exclusive.

Turning now to the elements which appear on the vertical axis of the table, there is first an overall distinction between sequential (the upper part of the axis) and non-sequential tenses (the lower part). Sequential tenses are used to indicate that some series of events falls into a temporal order of succession or simultaneity more complex than can be expressed by a simple three-fold distinction between past, present and future. The 'earlier than' and simultaneity relations (Diver uses 'before', 'after' and 'simultaneous') are introduced to cope with this sort of complexity. As a simple example, consider 'After he arrived the lecture began'. Diver characterises this use of '-ed' form as expressing that "the event indicated by the attached verb is one of a sequence of events that took place in the past" (p. 164). The sequential aspect is commonly indicated, he says, when the '-ed' form is accompanied by words that indicate a sequential relationship in time, such as 'before', 'after', 'earlier than', and 'later than', or when the form is embedded in a narrative, where the chronological unfolding of the story may indicate the ordering.

We find Diver more specific about the ordering when we ascend the vertical axis. The 'had -ed' form—corresponding to the traditional past perfect or pluperfect—is used in the context 'The lecture had begun when he arrived' to express that "the event indicated by the attached verb took place before another event in the past" (p. 165). Diver draws support for this proposal from, amongst other things, the suggestion that in 'The lecture had begun after he arrived', for example, "the incompatibility of 'before' (had -ed) with 'after' prevents 'arrived' from being the "other event" referred to in the definition" (p. 165). The arrival would serve as the event required by the definition in the earlier example 'The lecture had begun when he arrived', whereas in the present example there must be

some third event understood in the context. Such a third event is indicated by one meaning of the sentence 'They learned that the lecture had begun after he arrived', where the learning serves the required purpose of being the event before which the lecture begins.

To indicate that an event occurs simultaneously with some other event in the past the 'was -ing' form is used, as in 'He arrived when the lecture was beginning', and to indicate that an event occurs after some other in the past, two forms are available: 'would -O' and 'was -ing' again. The latter use of this last form is found with sentences in which the form occurs together with other words that are incompatible with the former meaning; for example, in 'It was twelve noon and the train was leaving at six', where the times indicated by the two verbs cannot be simultaneous. (This is what '+ non-simultaneous' means in the table where it occurs together with the form 'was -ing'). There are verb forms which can be used to express that an event occurs before, simultaneously with, or after some event in the present and in the future, as shown in the table.

As we have already indicated, these three elements of meaning within the categories of past, present and future tenses are related by "oppositions of exclusion" in much the same way as the elements on the horizontal axis and form a mutually incompatible trio. A similar situation obtains elsewhere on the vertical axis, and in fact several groups with mutually exclusive members occur. Diver frequently resorts to these incompatibilities when trying to secure the distinctions he introduces by arguing along the lines we have just seen with the 'was -ing' form. To do full justice to his analysis, we need more than two dimensions in our graph—a separate dimension, by analogy with the construction of the horizontal axis, for each group of mutually exclusive elements, together with an account of any further restrictions on permitted combinations and allowing the possibility that no element is selected from some groups. But for our purposes it will not be necessary to go into such detail, since we have not considered by any means all of the aspects of tense usage which Diver mentions, and the remaining section of the sequential part of the axis in our illustrative fragment should by now be reasonably clear without the need for any more complicated exposition. Enough has been said to illustrate the way in which the 'earlier than', 'later than' and simultaneity relations are explicitly introduced in conjunction with the three-fold distinction between past, present and future in Diver's scheme. This again reflects a position suggested in Chapter I that temporal indexicals alone are insufficient to give adequate expression to the ordering of events in time and that B-statements expressing that some moments are earlier than, later than or simultaneous with others, must be introduced.

Turning now to the lower part of the vertical axis and the non-sequential tenses, the important distinction Diver draws here is that between definite and indefinite tenses. The past indefinite, we are told, is expressed by the form 'have -ed' with the meaning that "the event indicated by the attached verb took place on an indefinite occasion in the past" (p. 156). The past definite, on the other hand, is expressed by

a now familiar form "was -ing' which is used in such a way that "the event indicated by the attached verb took place on a definite occasion in the past" (p. 157). Diver draws support for the soundness of this distinction in the first instance from a consideration of the restriction on the choice of tense forms available in combination with words such as 'ever' and 'whenever', which are normally used to indicate a time or times, but no specific moment. So we have, for example, 'No one has ever asked such a question before' and 'Whenever he has gone, he has been well received', where the definite form 'was -ing' cannot be substituted for the indefinite form since this would yield '*No one was ever asking such a question before' and '*Whenever he was going he was being well received'. (Following the custom amongst linguists, an asterisk immediately preceding a sentence is used to indicate that that sentence is deviant.) Similarly, with words or phrases normally used to indicate a specific moment in time, such as 'last Tuesday' and '4th December 1965', there are again restrictions on the choice of linguistic tense allowed. Thus, 'He was playing golf last Tuesday' is correct, but not '*He has played golf last Tuesday'; and if we consider the passive mood (on which Diver does not comment), 'The youngest son was born on 4th December 1965' occurs, but not '*The youngest son has been born on 4th December 1965'. The earlier example, '*He has played golf last Tuesday' can, however, be set in a larger context without deviance; for example, 'He has played golf last Tuesday and every Tuesday before that for several years'—but only at the expense of removing any suggestion of a reference to just one specific occasion. Finally, there are words which may be used to indicate moments in time and which may be used to refer either to some specific occasion or indefinitely to some occasions. Diver suggests that in combination with these words an indefinite past tense forces an interpretation of indefiniteness. Thus, 'He has played golf on Tuesday' cannot refer to a specific Tuesday, whereas the specific interpretation must be given in combination with a definite tense: 'He was playing golf on Tuesday' means a specific Tuesday.

We might comment at this point on the sort of linguistic unit that might appropriately be regarded as a tense in Diver's sense. On the one hand, it has already been suggested that such a tense comprises the inflexion of a verb together with any auxiliaries. But when discussing the use of the 'was -ing' form to express that some event occurs after some point in the past, there seemed to be a suggestion that the tense comprises not only the verb form, but also some means of expressing non-simultaneity. Precisely how the linguistic means of expressing non-simultaneity are to be delimited is not a question I want to take up here, but it does seem that Diver ought to provide some such account if his notion of a tense and his project of correlating linguistic forms with what they express are to be at all complete. Again, when discussing the definite/indefinite distinction other linguistic elements apart from verb inflexions and auxiliaries played a central role and perhaps should be included as part of the linguistic unit to be considered a tense in Diver's sense. Thus, '*He has played golf last Tuesday' is deviant because although the forms 'has -ed', 'has -ed... last Tuesday and every Tuesday before

that for several years' and 'was -ing... last Tuesday' are tenses, 'has -ed... last Tuesday' is not. Moreover, Diver claims that the definite past is commonly expressed by the '-ed' form, though this is not the only use of this form, and so we might take the view that linguistic units like '-ed... last Tuesday' should be regarded as tenses in Diver's sense. Certainly in the sequel we shall look beyond the inflexion of the verb and its auxiliaries to find what tense is expressed by the use of some of the sentences we shall consider.

The way in which Diver treats the '-ed' form as it occurs in the lower part of the vertical axis is rather strange. As with its occurrence in the sequential tenses, the use of the '-ed' form as a non-sequential tense is held to be unspecific with respect to the senses mentioned on the vertical axis. Diver explains on p. 153 that the '-ed' form is used to make reference to a time in the past, and later on on p. 158 that it is commonly used to indicate a definite time. But this form does not appear alongside the 'was -ing' form as a means of expressing a past definite tense, whereas the 'was -ing' form in the sequential part of the table does appear alongside the 'would -O' form as a means of expressing that an event occurs after some time in the past, as well as appearing by itself as a means of expressing that an event occurs simultaneously with some point in the past. If, as Diver seems to suggest, the '-ed' form is not specifically tied to the expression of either a definite or an indefinite tense, but may be used for both purposes, then as a matter of overall consistency we would expect the '-ed' form to appear in the table as a member of both the definite and the indefinite sections of the past column, and not to require a position of its own. Similar comments apply to the use of '-ed' as a sequential tense.

According to his program of regarding present and future tenses as analogues of past tenses, he goes on to consider the definite/indefinite distinction as applied to present and future tenses. In the latter case, Diver contends that the distinction finds no expression (p. 177), and he lumps together several verb forms he considers future tenses in the same undistinguished horizontal row in which the '-ed' form occurs in the past column. Later on I shall suggest that Diver's distinction between sequential and non sequential tenses collapses into one, when it will be possible to make with some plausibility the claim that something like the definite/indefinite distinction does apply to future tenses. For the moment, however, I shall go on to consider the way Diver applies the distinction to the present tense.

The application of the distinction here is not altogether a happy one. As an illustration we are presented with the "general truth" 'The sun rises in the east', which is contrasted with 'The sun is rising in the east'. The verb form '-O' in the former is classified as an indefinite present whereas the form 'is -ing' in the latter is classified as a definite present. Diver "freely renders" what is expressed by the present indefinite and definite somewhat paradoxically by: "The meaning indicated by the attached verb takes place on an indefinite (definite) occasion in the present" (p. 173). Let us suppose that the word 'meaning' is replaced by the word

'event' here. At one point Diver proposes to interpret "an indefinite occasion in the present" as "an inexplicitly indicated time, including the present" (footnote, p. 173), and the "general truth" is held to express that the event indicated by the verb occurs at a time or times manifesting such a characteristic. Now, it is clearly possible that, at some place, the sun is not (presently) rising in the east although the general statement is true, and so the event indicated by the attached verb does not necessarily take place "at some time including the present". But more importantly, it is hardly reasonable to say that the event referred to takes place at an "inexplicitly indicated time" just because the sentence is general with respect to time and so may be taken to say that, for any time t, if the sun rises at t, it does so in the east. The definite/indefinite distinction as applied to past tenses was not quite so vague as this, for there the distinction rested on whether the reference to time was to some specific moment or not, and not on whether the reference was explicit.

At the same time Diver gives what is in some ways a clearer characterisation of the distinction as applied to the present:

> The difference in meaning between 'past' and 'present' in this system is that 'past' means that the occasion indicated by the verb took place prior to the moment of speaking and 'present' means the occasion includes the moment of speaking.
>
> In relation to the definite-indefinite opposition the present differs from the past in that there is, for the present, only one definite moment, the moment of speaking. The definite occasion referred to by the 'is -ing' signal is therefore always the moment of speaking, and the definite occasion avoided by the '-O' signal is also the moment of speaking. (p. 173)

An indefinite present tense, it seems, makes reference to some time or times other than the moment of speaking but within the confines of a 'specious present'. But this is too strong. A definite present tense is used to make reference to the moment of speaking whereas an indefinite present tense need not necessarily be used to make reference to this moment, but merely to some time or times within a larger segment of time including the moment of speaking. The use of the indefinite present is not incompatible with the possible circumstances that the event indicated by the verb occurs at the moment of speaking; the point is that it makes no explicit mention of this fact. In reference to 'He sleeps in the morning' which may be used when the person referred to is actually sleeping, Diver says "It is just this lack of explicitness in reference to the present moment that characterises the indefinite present" (p. 173).

With this explication of the distinction as applied to the present, Diver adopts a procedure similar to that used to substantiate the application of the distinction to past tense in order to illustrate his point, namely, of producing examples where there seem to be some restrictions on the verb forms available. Thus, the indefinite form is compatible with expressions not specifically referring to the present moment, as with 'He sleeps in the morning', but this form is incompatible with an expression making reference to the present moment—for example, '*He sleeps at this

moment'. The definite form is better in the latter context, giving 'He is sleeping at this moment'. But then the definite form is incompatible with an expression which does not refer specifically to the present moment, so that we have '*He is sleeping in the morning'. Diver comments that this last sentence can be embedded satisfactorily in a larger context in which the period of time to be taken as the present is made precise (his words are "a period of time can be made definite by limiting it through opposition to a larger, indefinite period of time" (p. 173)), and he gives as an example 'He is sleeping in the morning this week because he has been put on the night shift'. It is clearer from this explication and these examples why Diver should regard general statements like 'The sun rises in the east' as present tensed rather than tenseless: he seems to take them as limiting cases of the present indefinite where the present has become so extended as to encompass the whole of time.

Now, it seems to me that this account is not consistent with that part of his account considered in the earlier part of this exposition, and also encounters problems when considered independently. All the problems arise from the distinction Diver draws between two senses of 'present': on the one hand, we have the present moment, identified with the moment of speaking, and on the other, we have a larger, more specious present which includes the moment of speaking. In the passage quoted above Diver proposes that 'past' be broadly interpreted in his system in uniformity with the latter sense of 'present', with the meaning "that the occasion indicated by the verb took place prior to the moment of speaking". But this is not the meaning of 'past' used in his account of sequential tenses. There the use of a sequential past tense is to relate the time at which the event indicated by the verb occurs to some moment in the past, so that to say that the 'would -O' form means 'past, after' is to say that the event indicated occurs after a moment in the past. Thus, 'past' means simply 'a particular moment before the present'. Furthermore, it is compatible with this meaning that the event does not occur earlier than the moment of speaking. Consider, for example, the sentence 'John said that he would come along after dinner', which we shall suppose is uttered after John has finished his dinner but before he arrives. What John said might still be true since he might well arrive after this statement was made, in which case the event indicated by the verb in question –John's arriving–does not occur in the past. Moreover, the thesis of the incompatibility of the elements 'past' and 'present' in reference to the same event can no longer be maintained. For if the present is to be understood as some period of time encompassing the moment of speaking, and the past as the period of time preceding the moment of speaking, it follows that there is a possibility of overlap so that some times are both past and present.

These are substantial problems for Diver since he does explicitly say on p. 165 that 'past' is to have the same interpretation in his analysis of both sequential and non-sequential tenses. But apart from these considerations of overall consistency, there still remains a final difficulty with his application of the definite/indefinite distinction to the present tenses. On the one hand, "there is, for the present, only

one definite moment, the moment of speaking. The definite occasion referred to by the 'is -ing' signal is therefore always the moment of speaking." But on the other hand, Diver in fact gives us an example in which the present definite is not used to refer to the moment of speaking, namely 'He is sleeping in the morning this week because he has been put on the night shift'. Here the definite moment in the present must include the moment of speaking as only a small part, so that the distinction between the two senses of 'present' collapses. Diver might possibly want to rebut this argument by allowing an even greater interval, including this particular definite moment in the present, to serve his purposes for interpreting the indefinite present tense. But having lost the criterion—the moment of speaking—for distinguishing the two senses of 'present', to have a specious present and in addition an even more specious present seems a little artificial. I do not mean to suggest that there is something wrong with the contention that there is just one definite moment in the present in a given context, or that this moment should have some finite duration—perhaps considerably longer than the time of utterance. (I have already indicated that I do not want to accept a reduction of the present tense in terms of the time of utterance). But one specious present is quite enough, and I do not want an even more specious present in addition.

Now, if there is to be just one present in the semantical analysis (with 'past', and 'future' for that matter, retaining their former interpretations), what can we say about the restrictions on tense forms available in the sort of context Diver presents? The so-called definite present presents no problem—this is used to refer to the present moment. As for what Diver called the indefinite present form '-O', in some contexts this looks very much like what might be described in more traditional terminology as a habitual tense. In fact, Diver has something very much like this, although not shown in our table since I will not have much to say about it in the sequel. He introduces a repeated tense, which he suggests is expressed by the form 'will -O' together with some extra-verbal indication of indefiniteness; for example, 'sometimes'. So we have 'Sometimes it will start and sometimes it won't', and 'Sometimes he will walk for hours at a stretch', where the events referred to are said to occur repeatedly. It seems that the single '-O' form will do the same work, so that we have 'Sometimes it starts and sometimes it doesn't', and 'He walks for hours at a stretch'. The standard example of the use of an indefinite present tense, 'He sleeps in the morning', is just like this, expressing a repeated occurrence. Now, the period of time throughout which the repetition occurs does include the present in these cases; but such an extended period of time is not what we mean by the element 'present' in the semantical analysis, and so the thesis of the incompatibility of 'past', 'present' and 'future' is not damaged. To deal with repeated tenses in a logical analysis, it seems to me that we need an operator which relates moments in the past, present or future to larger intervals which include these times by analogy with the way in which 'had -ed' is used to relate the event indicated by the verb by the 'earlier than' relation to a moment in the past. Then, just as 'had -ed' is said to have the meaning 'past, before', so '-O' might be said to express the meaning 'present, extended'.

Having said this, however, it seems to me that the habitual and continuous tenses present something of a problem. Consider, for example, a distinction Diver draws between an extended and a repeated past tense to reflect the difference between, say, 'He kept running' and 'He would run', and between 'He kept on hitting his wife' and 'He would hit his wife' respectively. It is suggested that an extended tense is used to indicate that an event occurs continuously throughout a relatively (for the context) large interval, whereas a repeated tense is used to indicate that something occurs intermittently. But it is not clear that this alone will serve to distinguish 'He kept on hitting his wife' and 'He would hit his wife', for it might be objected that since hitting seems to comprise a series of discrete actions, it is not the sort of thing that can occur continuously. This would then raise an analytical problem. For so far it has been assumed that an analysis of tenses can be pursued independently of the meaning of the verb. But the issue just raised suggests that this is not so, at least in the case of habitual and continuous tenses, and that the analysis must penetrate more deeply into the structure of the predicate so that we can distinguish verbs like 'to hit', which are used to describe occasions comprising discrete events, from verbs like 'to run' which do not seem to describe actions which comprise non-continuous events. The analysis developed in the sequel is founded on this assumption that the analysis of tenses can be pursued independently of the meaning of the verb or predicate, and in order to remain within the bounds of what is justifiable under this assumption, and in particular not to beg any questions over the issue just mentioned, habitual and continuous tenses will not receive much attention.

Statements that are general with respect to time, like 'The sun rises in the east' and 'Whenever sodium salts are subjected to the flame test, they produce a yellow colour', do not present this problem. They can be taken as tenseless statements of the form 'For all times t, if A at t then B at t'. The range of the quantifier, being the whole of time, does of course include the present moment as one possible value of the variable. But since it includes all other times as well there seems to be no reason for regarding such general statements as present tensed rather than past or future tensed; and once this is seen, there does not seem to be any reason for regarding them as tensed at all. Now, there may well be other pragmatic aspects relevant to these general statements apart from indications of whether something is past, present or future. In particular, the range of the quantification over moments may be determined by contextual features in much the same way as the range of quantification over individuals is often restricted to, for example, a certain class of people. Thus, there may be times when it is not the case that the sun rises in the east: for example, before the Earth was formed, or when the Earth ceases to rotate on its axis; but such extremities of the temporal manifold lie outside the domain of the quantifier underlying 'The sun rises in the east'. And since distinctions of past, present and future would be drawn within the domain determined by the context, the thesis that such general statements are tenseless, rather than involving some gigantic present moment, can be maintained.

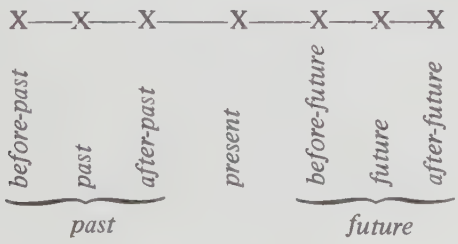
This concludes the exposition of what, for present purposes, are the salient points of Diver's paper. I shall go on to consider Reichenbach's analysis of tenses and try to characterise some of the distinctions drawn here more perspicuously—for example, that between definite and indefinite tenses—by way of suggesting improvements to Reichenbach's analysis. I shall also want to drop the distinction between sequential and non-sequential tenses; or at least, to drop the distinction as Diver draws it. Finally, I would like to add that I think it a mistake of Diver's to take the past tenses as his prototype and consider the strength of the analogy between these and present and future tenses. The present column provides the axis of symmetry so far as the semantical system is concerned, and I would have thought it far more useful to take this as the prototype and, where there are differences, to consider whether past tenses are like the future tenses in this respect. Looking at tenses in this way would have made the application of the definite/indefinite distinction to the present far less plausible in the first place.

3. Reichenbach's Analysis of Tenses

Reichenbach's characterisation of tenses as token-reflexive expressions was mentioned in Chapter I and rejected there as a means of eliminating tensed in favour of tenseless statements. Apart from the token-reflexive aspect of his analysis, however, Reichenbach's views on tenses contain some sound points which are quite close to some of Diver's views and illuminate much of what Diver has to say about tenses. Moreover, as I shall go on to show in later chapters, the analysis is far from "a hindrance rather than a help in the construction of a logic of tenses"[4] as Prior would have it.

Reichenbach's 'three-place structure' of tenses derives from Jesperson's analysis of such tenses as the pluperfect and the future perfect. In *Essentials of English Grammar,* Jesperson suggests that the three-fold distinction between past, present and future is not enough:[5]

> Under each of the two divisions of infinite time we may refer to some point as lying either before or after the main point of which we are actually speaking. In this way we get the following seven points:



[4] Prior, *Past, Present and Future,* p. 13.

[5] O. Jesperson, *Essentials of English Grammar* (London, 1933), pp. 230–231.

The resemblance with Diver's account is already apparent.

Rather than talk of "the main point of which we are actually speaking", Reichenbach introduces the notion of a 'reference point'. The reference point may or may not coincide with the time of utterance or the time at which the event described occurs, so he further distinguishes a *point of speech* and a *point of the event*. To give a simple illustration of his analysis, a sentence like 'Peter had gone' is used to say that the point of the event, when Peter went, occurs at some time before the point of reference, which is some time between this time and the point of speech. Which time point is taken as the point of reference is not clear from an isolated sentence like the one given, and Reichenbach goes on to emphasise the importance of the context in which the sentence occurs in determining the reference point. In narratives, for example, the past tense indicates that the point of reference occurs in the past, and the reference point remaining fixed throughout several sentences, it is possible to order the events described in the neighbourhood of this point into some sort of sequence. Reichenbach quotes a passage from W. Somerset Maugham's *Of Human Bondage* as an illustration:

> But Philip ceased to think of her a moment after he had settled down in his carriage. He thought only of the future. He had written to Mrs. Otter, the massiere to whom Hayward had given him an introduction, and he had in his pocket an invitation to tea on the following day.[6]

The point of reference is the time when he was thinking "only of the future". Some of the events described in the passage occurred earlier than this, like the settling down in the carriage, the writing of the letter and the giving of the introduction, and are related in the past perfect. Sometimes the reference point is more explicitly indicated: for example, Reichenbach quotes a passage from Macaulay beginning "In 1678 the whole face of things had changed. . .", where the reference point is the year 1678; and again, we might begin "Ten years ago things were much different. . .". Much more vaguely, a reference point is indicated by "Once upon a time. . .". Between these extremes the time at which a certain event occurs may be used as the reference point and the word 'when' often serves to indicate this, as in 'When John arrived I had already left'.

With this three-point structure, allowing that the points may sometimes coincide, Reichenbach provides his analysis of the more simple tense constructions in the following table (p. 290. 'S' stands here for the point of speech, 'R' for the point of reference and 'E' for the point of the event):

Past Perfect	*Simple Past*	*Present Perfect*
I had seen John	I saw John	I have seen John
—X—X—X—	—X——X—	—X——X—
E R S	E,R S	E S,R

[6] Hans Reichenbach, *Elements of Symbolic Logic* (New York, 1947), p. 288. All other references to this work in the present section are indicated in the body of the text.

Present	*Simple Future*	*Future Perfect*
I see John	I shall see John	I shall have seen John
——X——	——X——X——	——X—X—X——
S,R,E	S,R E	S E R

"Actual language", Reichenbach goes on to say, "does not always keep to the schemas given in our tables" (p. 292), and he describes some cases of divergence. It is by now a familiar point that particular verb forms are not always used to express the same sense. Several examples illustrating this point were given at the beginning of the chapter, and Diver has expressed the point by sometimes entering the same word form in more than one place in his table. Our interest in Reichenbach's analysis lies more with the structures he suggests are expressed by English tense forms, than with the rigid correlation between particular word forms and these structures.

Having considered these more elementary tense constructions, Reichenbach then goes on to discuss more complex sentences where the tenses in various clauses are adjusted to one another by rules for what grammarians call the *sequence of tenses*. But before getting on to this particularly interesting aspect of his analysis (which is postponed until Chapter IV), I would like to consider criticisms of this scheme raised by Prior which will lead to a better understanding of the function of reference points.

4. Prior's Objections to Reichenbach's Analysis: The use of Indexical Expressions and "the Nature of Presentness"

Prior considers Reichenbach's analysis just after having considered and rejected Smart's account of tenses,[7] according to which past, present and future tenses mean that the events mentioned occur earlier than, simultaneously with, and later than the utterance with which they are described, respectively. There is a possibility, so Prior suggests, that the difficulties he sees in Smart's analysis arising from certain considerations of the future perfect tense might be dispelled if Reichenbach's more comprehensive account can be sustained. For Reichenbach also wanted to maintain a token-reflexive account of tenses, as we have seen in Chapter I—hence the introduction of the point of speech.

Prior, antagonistic to any account of tenses in the spirit of the Russellian analysis, sets out to discredit Reichenbach's scheme by suggesting, in the first instance, that it is too simple. For it is possible to construct more complicated tenses than the future perfect, although not commonly used, which would require, so Prior maintains, two reference points. The example cited is 'I shall have been going to see John', which Prior represents by the diagram:

——— X ———	X ———	X ———	X ———
S	R2	E	R1

[7] Prior, op.cit., pp. 12–15.

(There might be other possibilities since E can occur simultaneously with or later than R1 as well.) Because Reichenbach's account falls short of the general situation where two or more reference points may be required, Prior thinks it "unnecessary and misleading to make such a sharp distinction between the point or points of reference and the point of speech; the point of speech is just the *first* point of reference".[8] Prior suggests that this move destroys Reichenbach's way of distinguishing between the simple past and the present perfect, and makes pastness and futurity always relative to some point of reference, though not necessarily the first (i.e. the point of speech). This then leads him to the comment that Reichenbach's views were more of a hindrance than a help in the development of tense logic, misconstruing the nature of presentness and failing to see that there is a future presentness and a past presentness which leads to the generation of the hierarchical structure described in Chapter I.

Now I have already argued against the reduction of tensed to tenseless statements, and in particular against Reichenbach's proposed definition of 'now'. And so for this reason I am quite happy to adopt Prior's simplification of regarding the point of speech on a par with the point or points of reference. But I do not see that Prior has established that the scheme must be generalised along the lines he advocates. It is pertinent to point out at this stage that Prior also held the view that 'now' is eliminable, though not of course according to Reichenbach's plan. This is his "redunancy theory" of the present tense, which he expresses as the claim that the proposition 'It is (now, at present) the case that p' is the very same proposition as the proposition p.[9] The present tense, on this theory, is the understood tense of any tensed proposition that has no further specified tense; hence, all atomic propositions are viewed as present tensed. Moreover, if tenses are operators which uniformly construct tensed propositions from tensed propositions, then every tensed proposition must have at least the present tense. A formula of the kind Pp in Prior's system, for example, is read 'it is the case that it was the case that it is the case that p', asserting the present pastness of p's presence. From this we can now construct FPp, read as 'it is now the case that it will be the case that it is the case that it was the case that it is the case that p', asserting that the present pastness of p is relative to some future time–itself presently future present! Since every tensed proposition must be present tensed in this way, there is no need, so the theory goes, to introduce an operator for 'it is now the case that p' as the plain p will do.

It has been objected that this theory does not do justice to the way 'now' behaves in idiomatic English. But Prior retorts that the tense-logician is naturally reluctant to countenance an additional present tense to cope with 'now', even if it were compatible with the one outlined above. His doubts on this score at one time apparently forced Prior into the position of maintaining that the tense-logician

[8] Prior, *Past, Present and Future*, p. 13.

[9] Prior, *Papers on Time and Tense* (Oxford, 1968), Chapter 2.

must relinquish any hope of introducing another operator for 'now'.[1] But his qualms over this matter have since been settled by discussion with Hans Kamp.[2] In spite of these results on the technical side, however, Prior was still reluctant to introduce another present tense and maintained his thesis of the eliminability of 'now'.[3]

The situation here is similar to that encountered in the previous section in connection with Diver's two senses of the present. The conclusion there was in favour of just one sense of the present, and in the same vein I fully sympathise with Prior's reluctance to endorse two present tenses. However, I prefer to relinquish the present tense he maintains tense-logicians must have—i.e., the present which may be future present, past present, and so on. Far from being redundant as in Prior's scheme, it seems to me that we can do more justice to the present tense and the word 'now' by explaining its function as that of referring to the point of reference in the present. Thus, taking Prior's initial point, the point of speech is to be regarded as a point of reference on a par with the others, except that in this case it is the present moment.

What now of the other reference points? There is in Prior's criticism the suggestion that there may be considerably more than one point of reference in the future, which, together with the claim that what we have so far destroys Reichenbach's way of drawing the distinction between the simple past and the present perfect, seems to suggest that Prior thought that the function of the reference points is assimilated under his generalisation to that of the point of the event. So not only is the point of speech the same kind of thing as the points of reference; reference points are, in his view, also on a par with the point of the event and there are no interesting distinguishing characteristics of the various times required in the analysis of the more complex tenses. However, Prior has hardly given any grounds for supposing that his example, 'I shall have been going to see John', involves two reference points in the future. For the essence of Reichenbach's points of reference is that they serve in some context to provide a relatively fixed point about which the events related may be ordered. But Prior provides no such context himself, and it is difficult to see how such a context could be constructed in which there would be any point in having two reference points as he suggests. Moreover, only the reference point R1 need occur in the future: the time R2 need not necessarily lie between S and R1; it is enough that R2 be before R1. Remembering Diver's analysis, a better account of the statement in question would be to assume one reference point in the future—this is why it is a future tensed statement—and to say that the event occurs after some time before this reference point. Diver does not actually consider this particular possibility, but it is clearly a natural extension of his scheme and so no more than an omission on his part.

Just as the indexical expression 'now' serves in its frequent occurrences to pick

[1] Prior, '"Now"', *Nous*, 2 (1968).

[2] Kamp's account of 'now' is discussed in Chapter V.

[3] Prior, ibid.

out the reference point in the present, so the word 'then' often serves to pick out the reference point in the past or the future. 'Then' is ambiguous in this way. Consider the passage of Macaulay's quoted by Reichenbach (pp. 288–9):

> In 1678 the whole face of things had changed. . . eighteen years of misgovernment had made the. . . majority desirous to obtain security for their liberties at any risk. The fury of their returning loyalty had spent itself in its first outbreak. In a very few months they had hanged and half-hanged, quartered and emboweled, enough to satisfy them. The Roundhead party seemed to be not merely overcome, but too much broken ever to rally again. Then commenced the reflux of public opinion. The nation began to find out to what a man it had entrusted without conditions all its dearest interests, on what a man it had lavished all its fondest affection.

The year 1678 is the point of reference here. Events described in the first part of the passage occur before this time and are related in the past perfect, whilst events related in the final two sentences occurred during 1678 and are related in the simple past. The penultimate sentence begins with the word 'then', and it is clear that the time thus indicated is both the time during which the events related in the simple past occur and the year 1678. Whether the reference point indicated by 'then' is in the past or the future is usually made clear by the context. Thus, we have in the statement 'When she arrives it will be dark, but I hope that the light will be repaired then' an example of the use of 'then' to pick out a time in the future.

In this way we have an elementary account of the indexical character of tenses. 'Now' serves to refer to the present moment and 'then' to a point in the past or the future. This matches Diver's three-fold division of tenses into past, present and future. The sequential tenses are used to indicate that events occur before, simultaneously with, after, and so on, one of these three points. This is the idea underlying the logic of tenses I shall present in the next chapter.

Two points should be made at this stage. First, it is sometimes said that 'then' can serve as part of a tensed conjunction 'and then' or 'and next'–for examle in von Wright's paper '"And Next"' and elsewhere.[4] It would appear that the occurrence of 'then' in the passage from Macaulay above has this meaning. But there need not be any incompatibility here, since the 'and then' sense is conveyed by the transition from the use of the past perfect to describe events occurring before the time referred to by 'then'. There are cases when both interpretations are not satisfied in such a straightforward manner; consider, for example, 'He will get up, shave and eat his breakfast before going to work'. If this is to be paraphrased as 'He will get up and then shave and then eat his breakfast and then go to work', not every occurrence of 'then' refers to a reference point in the future. But if we take it that the first clause is in the simple future and so fixes the reference point, then the remaining clauses can be regarded as expressing Jesperson's 'after

[4] G.H. von Wright, '"And Next"', *Acta Philosophica Fennica*, Fasc. 16, (1965); J. Clifford, 'Tense Logic and the Logic of Change', *Logique et Analyse*, No. 34 (1966); G.E.M. Anscombe, 'Before and After', *Philosophical Review*, 73 (1964).

future'—which he claimed had practically no grammatical realisation—and so describing a sequence of events occurring later than the reference point.

Secondly, 'then' resembles 'now' in so far as no matter how oblique the context in which it occurs, the time to which it refers is unaffected by whatever operators appear to encompass it within their scope. Prior compares the behaviour of 'now' to that of the word 'any', as opposed to 'every', in taking the whole sentence as its scope, and 'then' behaves in much the same way. Thus, 'You're not right at any time' is not the denial of 'You are right at any time (the denial would be 'You're not always right'); rather, it is equivalent to 'At all times you are not right', the word 'any' indicating that the negation must fall within the scope of the universal quantifier. Similarly, 'John said he would be in London now' does not mean that John stated "I am in London", but rather that the time John said he would be in London is the present moment. In general, no matter how deeply embedded within a sentence, 'now' refers to the point of time from which the whole sentence is viewed. This is one way in which our principle that the point of view of the speaker dominates all subordinate contexts finds expression. 'Then' also dominates all subordinate contexts and takes the whole sentence in which it occurs as its scope. This is clearly the case with the example already given above, 'It will be dark when she arrives, but I hope that the light will be repaired then', where 'then' refers to the same time as that indicated by the future tense in the first clause, although the word seems to occur within the scope of the phrase 'I hope that' which is in the present tense. We might express the principle that the point of view of the speaker dominates all subordinate contexts by saying that the way in which time is divided into past, present and future is the same for all subordinate contexts as it is from the point of view of the speaker.

So far I have said something about Reichenbach's point of speech and point of reference, but nothing as yet of the interpretation of the point of the event. According to Reichenbach the point of the event sometimes coincides with a reference point, in which case we get the simple past and the present tenses, and sometimes not, in which case we have, amongst other things, the simple future and the past, present and future perfect tenses. So it is really only in the latter cases that the distinction between the reference point and the point of the event comes into its own. Now in Diver's scheme the forms 'had -ed', 'have -ed' and 'will have -ed', corresponding to Reichenbach's past, present and future perfect tenses, are said to express that the event indicated by the attached verb takes place before a moment in, respectively, the past, the present and the future. But notice also that the 'have -ed' form occurs elsewhere in the table, notably as the past indefinite. These two uses of the 'have -ed' form are not, it seems to me, distinct, and we can identify the function of this form in both cases so that the 'present before' is also the past indefinite, and this can then be identified with the present perfect. The analysis of the present perfect then becomes 'the point of the event is some indefinite time before the present moment', and it remains to say in what the indefiniteness of the point of the event consists. Diver suggests that the use of the indefinite past is to make

reference to possibly several occasions (an unspecified number) or to unspecified times. This looks very much like existential quantification, and I propose that instead of *the* point of the event we introduce existential quantification over times earlier than the present moment when the event or events are said to occur. Similarly, the past perfect is used to express that some event(s) occurs at some time(s) earlier than the point of reference in the past, and analogously for the future.

In this way the three elements 'past', 'present' and 'future' of Diver's system become the three reference points of Reichenbach, and the indefinite tenses are characterised by existential quantification along the lines just mentioned. The definite past tense can, with this rationale, be identified with the 'past, simultaneous', and then with Reichenbach's simple past when the point of the event coincides with the point of reference. We might note in this connection that Diver's 'was -ing' form appears both as the past definite and the 'past, simultaneous'. This procedure now suggests a way in which the definite/indefinite distinction may find application to the future tense: the future definite is used to express that the event described by the verb occurs at the point of reference in the future, whereas the future indefinite expresses that some event(s) occurs at some time(s) later than the present moment. The present definite is now just the present of Reichenbach, where the event described is said to occur at the present moment. There is no room for an indefinite present in this scheme.

Chapter III

A Logic of Tenses and Dates

1. Preamble

In this chapter the semantics of a formal system concerned principally with that aspect of propositions making reference to positions in time is developed. The system is motivated by the discussion of tenses in the preceding chapter; but considerations of whether the system does justice to the intricacies of our usage of tenses is not the only criterion of adequacy of interest here. In the everyday discourse of scientist and layman alike, the use of both tenses and dates in making reference to positions in time is firmly integrated, and it will also be required of the system that it goes some way in reflecting both these aspects of temporal reference. In particular, systems of interest here will encompass the views on the relation between tensed and tenseless statements expressed in the first chapter. Just how far we can go in meeting both constraints—of articulating both tensed and tenseless statements within a unified framework—and issues raised by the procedures required for this project, is amongst the most important concerns of the present investigation. In later chapters I shall try to bring out how successful our system is in this respect and draw attention to some issues raised by the procedures adopted.

It is evident from the discussion in the first two chapters that the series of moments in time must be represented in the semantics of the system as an ordered set. For this purpose a binary structure $(\mathbf{T}, <)$ is introduced, where **T** is a set (the set of moments) and $<$ is an order relation on **T**. We shall take it that $(\mathbf{T}, <)$ is a linear ordering; i.e., $<$ is asymmetric, transitive and connected on **T**. Some moments are to have a special status in our analysis, and three select elements are chosen from **T** for this purpose: **n** for the present moment, **p** for the point of reference in the past and **f** for the reference point in the future. Thus we have a system $(\mathbf{T}, <, (\mathbf{p},\mathbf{n},\mathbf{f}))$. Together with this structure a domain **D** of individuals and a set of relations defined on **T** and **D** for the interpretation of predicate letters of the formal language are introduced. Now, what can be said at this point about the nature of the object language which would be useful for our purposes in expressing relations amongst these elements of the semantic structure?

Consider a simple sentence in traditional subject-predicate form, say 'John plays golf'. This is expressed in logical notation in the form Aa, where $A_$ stands for the predicate '_ plays golf' and a for 'John'. $A_$ can reasonably be called a tensed predicate since it indicates the time at which 'plays golf' is true of John by

the use of the present tense. '_ plays golf', we want to say, is the sort of predicate which holds of whatever it might apply to at some time or other. Remembering now the discussion in Chapter I, this feature of holding of whatever it applies to at a time is by no means characteristic of all properties. 'Is earlier than', for example, is a case in point and does not hold between a pair of moments at any moment in time. Consequently, if we are to be able to express that some moment α is earlier than another moment β—we shall use $E\alpha\beta$ for this purpose—in the same system in which we are to express 'John plays golf', it will not do to express 'plays golf' simply as $B_$. '_ plays golf', in contradistinction to $E_,_$, is to be the sort of predicate which holds of an individual at a time, and this we shall make explicit by expressing 'plays golf' in the form ${}^{\alpha}B_$, where α is a variable ranging over moments in time. So we have that ${}^{\alpha}B_$ is a well-formed part of the tensed predicate $A_$ in which the moment variable α must refer to the present moment—i.e. to the select element $\mathbf{n} \in \mathbf{T}$—since $A_$ is present tensed. This we shall do by expressing $A_$ in the form $\Delta^{\alpha}{}^{\alpha}B_$, where the operator Δ^{α} binds all occurrences of α within its scope and is interpreted in such a way that these subsequent occurrences of α then refer to the present moment **n**.

To give for the moment just one more illustration of the formulation of a tensed statement, consider 'John has played golf'. This would appear to be an example of the present perfect which, according to our modified version of Reichenbach's scheme, is to be interpreted by means of an existential quantification over moments earlier than the present moment. Again using ${}^{\alpha}B_$ for '_ plays golf', we get the predicate in the present perfect tense by ensuring that the moment variable α is bound by this existential quantifier, and so the statement becomes $\Delta^{\beta}\mathrm{V}\alpha(E\alpha\beta \,\&\, {}^{\alpha}B\mathrm{a})$.

Tenseless dated statements are easily rendered once constants are introduced referring to fixed points in time—we shall call them dates. Thus, if *m* is such a constant standing for some time, say 2nd February 1970, then the tenseless dated statement 'Bertrand Russell dies on 2nd February 1970' is expressed in the form ${}^{m}Aa$.

The formulation of tensed statements just illustrated differs in at least one important respect from Prior's which is worth mentioning now. Prior developed his systems from considerations of modal logics, and his tense operators *F*, *G*, etc., are analogues of the monadic modal operators of necessity and possibility in the modal systems of Lewis and others. Accordingly, variables ranging over moments do not appear in the object languages he uses in the formulation of tensed statements; moments appear essentially only when semantic interpretations of tense operators are constructed. They appear in our object language initially because we want to draw a distinction between properties which hold true of the objects to which they apply relative to some moment in time and those which do not. (As we shall see, this is not a distinction which Prior expresses.) Now David Lewis[1] has shown that modal notions can be expressed in a first-order language

[1] David Lewis, 'Modal Logic and Counterpart Theory', *Journal of Philosophy*, 65 (1968).

by outlining a translation procedure according to which all well-formed formulae of modal logics with monadic modal operators can be translated into well-formed formulae of predicate logic. The first-order language corresponding in this way to a modal logic is sometimes called the modal analogue of that logic, and in the modal analogue variables ranging over possible worlds appear explicitly in those formulae expressing modal concepts together with the 'accessibility relation'[2] defined on the set of possible worlds. In the case of tense logics in the spirit of Prior, the accessibility relation appearing in the semantics of these systems is interpreted as the 'earlier than' relation, and possible worlds as moments in time, so that statements relating to the 'earlier than' relation appear in the metalanguage whilst tensed statements occur in the object language. It would appear, then, that for our purposes a first-order language in the style of David Lewis' modal analogues, in which we can express both tensed and tenseless reference to time in the object language, is a more suitable tool than a language in the style of the Lewis modal systems. A first-order language of this kind is developed in the remainder of this chapter and elaborated in the next.

(It is true that Prior has considered dated statements within his systems. But the addition appears very much as an after-thought with, as I think, several implausible consequences; for example, dated statements seem to be construed as tensed rather than tenseless statements. Prior's views on these matters will be considered later, after a comparision between his systems and the one presented here on the formulation of tensed statements. The point of the decision just made on the choice of formal language is to provide a framework within which both tensed and tenseless statements can be expressed from the outset without overdue initial prejudice in favour of one kind of statement or the other.)

2. The Language L*

The family of languages L* outlined in this chapter will be elaborated somewhat in the following chapter so that L* is a subset of what we shall eventually take to be our language L.

Our first-order language is to be a two-sorted system containing individual variables and constants together with moment variables and constants. The language is determined by a set **Pred** of predicate letters, a set **X** of individual constants and a set **Y** of moment constants or dates, and is denoted by L*(**Pred,X,Y**), which will usually be abbreviated simply to L*, leaving the dependence on **Pred, X** and **Y** as understood. An n-place predicate letter is denoted by P^n_j, the subscript serving to distinguish the various n-place predicate letters, and again we shall sometimes use upper case Roman letters without indices as abbreviations. Elements from **X** will be denoted by *a, b* and *c*, with indices if necessary, and *m* is

[2] For accounts of the 'possible world' semantics for modal logics, see the works of Kripke, Hintikka and Montague cited in the Bibliography.

usually used to denote an element of **Y**, with indices if necessary. The language determined by **Pred**, **X** and **Y** can now be defined.

D1 The primitive symbols of L*(**Pred**,**X**,**Y**)

The primitive symbols of L*(**Pred**,**X**,**Y**) are the elements of **Pred**, **X** and **Y** together with

(i) for each positive odd integer i, a moment variable α_i, and for each positive even integer j, an individual variable x_j;
(ii) the connectives $\sim$ and **v**, and the universal quantifier Λ
(iii) the predicate letters E and $=$;
(iv) the "indexical quantifiers" Π, Δ and Γ; and
(v) left and right brackets (and).

Individual variables and constants are referred to indiscriminately as individual terms, and moment variables and constants as moment terms. The members of **Pred** are to be interpreted as predicates ascribing properties which hold of individuals at a time, and our terminology according to which a member P_j^n of **Pred** is called an n-place predicate letter is somewhat unusual in view of the formation rules determining which formulae are in L*(**Pred**,**X**,**Y**). Given this interpretation of the members of **Pred**, it might be thought appropriate to introduce another set of predicate letters into L* which are used to represent properties which do not hold of individuals relative to a time. In particular, the predicate letters E and $=$ might be thought to come from such a set, since they are not of the same general form as members of **Pred** (as will become apparent from the formation rules). However, E and $=$ are mentioned in clause (iii) of D1 since they are common to all the languages of interest here, and hence an integral part of the definition of the function L*. Although in the following chapter we shall allude to the availability of a set of tenseless or timeless predicates when real numbers are introduced, this development will not play an important role in the sequel and so is not formally catered for here. It will be fairly clear what steps should be taken on the formal side to allow for such timeless predicates.

We shall in fact go on to extend the language L* and the corresponding semantics developed here. But since there has not as yet been any discussion of the considerations motivating the introduction of more general atomic formulae in the extended language, to be called L(**Pred**,**X**,**Y**), this development will be left until the following chapter.

D2 *Formation rules for* L*(**Pred**,**X**,**Y**)

(i) The atomic formulae of L*(**Pred**,**X**,**Y**) are those of the kind (1) ${}^{u}P_j^n\, u_1 \ldots u_n$, where P_j^n is in **Pred**, u is a moment term and $u_1, \ldots, u_n$ are individual terms, (2) Eu_1u_2, where u_1 and u_2 are moment terms, and (3) $u_1{=}u_2$, where u_1 and u_2

are moment terms or individual terms. All atomic formulae are in **L*(Pred,X,Y)**.

(ii) If A and B are in **L*(Pred,X,Y)**, so are $(\sim A)$ and $(A \mathbf{v} B)$.

(iii) If A is in **L*(Pred,X,Y)** and contains $a \varepsilon \mathbf{X}$ but not the individual variable x, then $\Lambda x(x/a)A$ is in **L*(Pred,X,Y)**, where $(x/a)A$ is the formula which results from A by replacing all occurrences of a by x.

(iv) If A is in **L*(Pred,X,Y)** and contains $m \varepsilon \mathbf{Y}$ but not the moment variable α, then $\Lambda\alpha(\alpha/m)A$, $\Pi^{\alpha}(\alpha/m)A$, $\Delta^{\alpha}(\alpha/m)A$ and $\Gamma^{\alpha}(\alpha/m)A$ are in **L*(Pred,X,Y)**.

(v) A finite sequence of primitive symbols of **L*(Pred,X,Y)** is a formula in **L*(Pred,X,Y)** only if that is required by rules (i)–(iv) above.

An individual or moment variable v occurring in a well-formed formula (wff) A is said to be *bound in A* if it occurs within a well-formed part of A of the form $\Lambda v B$, $\Pi^{v} B$, $\Delta^{v} B$ or $\Gamma^{v} B$, and any occurrence of v within these subformulae is said to be a *bound occurrence of v in A*. Any occurrence of v in A which is not a bound occurrence is said to be a *free occurrence of v in A*, and v is said to be *free in A* if there is a free occurrence of v in A. A variable v_j is said to be *free for* v_i *in A* if there is no free occurrence of v_j in A falling within a subformula of A of the form $\Lambda v_i B$.

If u is a moment term which is either a date occurring in A or a variable free in A, this is sometimes expressed as ${}^{u}A$. Similary, if u is either an individual constant occurring in A or an individual variable free in A, this is sometimes expressed as Au. We shall generalise the substitution operation on wffs already introduced so that $(u/v_1,\ldots,v_n)A$ is the formula which results by replacing all occurrences of the terms $v_1,\ldots,v_n$ by u. The notation $A(B_1,\ldots,B_n)$ is used to indicate that $B_1,\ldots,B_n$ are subformulae of A.

Some abbreviations used in the sequel are the logical connectives $\mathbf{\&}, \supset, \equiv$ defined as usual and $\mathrm{V}v$ defined as $\sim\Lambda v\sim$. In addition, w,x,y and z are used as metalinguistic variables for individual variables, and α,β,γ and δ as metalinguistic moment variables. The outermost brackets of formulae are always omitted and sometimes the convention that the binding power of the constants decreases in the following order

$$\Pi,\ \Delta,\ \Gamma,\ \Lambda,\ \mathrm{V},\ \sim,\ \mathbf{\&},\ \mathbf{v},\ \supset,\ \equiv$$

is adopted to reduce the number of brackets where this assists the reading of formulae. As is by now apparent, quotation marks around expressions of L* are almost always omitted since this does not lead to confusion.

Finally, let me draw attention to an important feature of this system, that, according to clauses (iii) and (iv) of D2, quantifiers occur in wffs only when they bind variables within their scope, so that $\Lambda x_6 P_1^2 x_2 x_4$, for example, is not well-formed. Nor, for that matter, is $\Lambda x_2 \Lambda x_2 P_1^1 x_2$. The relevance of this restriction of the size of **L*(Pred,X,Y)** is discussed later.

3. Semantics for L*

The fundamental notion in the model theory is that of an interpretation, which is a structure comprising elements serving as referents for the non-logical constants of the language.

D3 *Interpretation for a language* L*(**Pred,X,Y**)

An interpretation **M** for a language L*(**Pred,X,Y**) is a 4-tuple (**D,T,(p,n,f)**, **V**), where

(i) **D** is a non-empty set (the set of individuals);
(ii) **T** is a (non-empty) set disjoint from **D** (the set of moments);
(iii) **p,n,f**ε**T** such that **p**<**n**<**f**, where < is the extension of E (see clause (iv,3) below);
(iv) **V** is a valuation assigning an interpretation to the elements of **Pred**, **X** and **Y**, and to E and = as follows:
1) If $a\varepsilon$**X**, **V**(a) is a fixed element of **D**;
2) If $m\varepsilon$**Y**, **V**(m) is a fixed element of **T**;
3) **V**(E) is a linear order relation < defined on **T** with the minimal properties of asymmetry, transitivity and connectivity;
4) **V**(=) is the identity relation defined on **D**$\cup$**T**; and
5) If P_j^n is in **Pred**, **V**(P_j^n) is a subset $\mathbf{R}_j^n$ of **T**x**D** .

Let **Z** be the set of denumerably infinite sequences $\mathbf{s} = (\mathbf{s}_1,\mathbf{s}_2,\mathbf{s}_3,\ldots)$ of elements of **T** and **D** such that the sub-sequence $\mathbf{s}_t = (\mathbf{s}_1,\mathbf{s}_3,\mathbf{s}_5,\ldots)$ comprising the odd members of **s** contains just members of **T** and the sub-sequence $\mathbf{s}_d = (\mathbf{s}_2,\mathbf{s}_4,\mathbf{s}_6,\ldots)$ comprising the even members of **s** contains just members of **D**. Following Mendelson[3], we shall define what is meant by a sequence **s** of **Z** satisfying a formula of L* under an interpretation **M**. As a preliminary step, the notion of an assignment **s***, determined by a sequence **s**, is defined, which is a unary function taking individual terms into **D** and moment terms into **T**.

D4 *Assignment*

An assignment **s*** determined by a sequence **s** of **Z** is defined by

$$\mathbf{s}^*(u) = \begin{array}{l} \mathbf{s}_i \text{ if } u \text{ is } x_i \text{ or } \alpha_i \\ \mathbf{V}(u) \text{ if } u \text{ is in } \mathbf{X} \text{ or } \mathbf{Y} \end{array}$$

If v_i is a moment variable and r a member of **T**, or v_i an individual variable and r a member of **D**, then $(r/v_i)\mathbf{s}$ is a sequence like **s** except that at the ith place r replaces $\mathbf{s}_i$. We are now in a position to give the inductive definition of a sequence satisfying a formula of L* under an interpretation **M**.

[3] Elliott Mendelson, *Introduction to Mathematical Logic* (New York, 1964), pp. 49–54.

D5 *Satisfaction*

$\mathbf{M} \models A$ [s] is read 's satisfies A under $\mathbf{M}$', where A is in $\mathbf{L^*(Pred,X,Y)}$, $\mathbf{M}$ is an interpretation for $\mathbf{L^*(Pred,X,Y)}$ and $\mathbf{s} \in \mathbf{Z}$, and is defined thus:

(i) $\mathbf{M} \models {}^{u}P_j^n\, v_1 \ldots v_n$ [s] iff $(\mathbf{s}^*(u), \mathbf{s}^*(v_1), \ldots, \mathbf{s}^*(v_n)) \in \mathbf{R}_j^n$;

(ii) $\mathbf{M} \models u = v$ [s] iff $(\mathbf{s}^*(u), \mathbf{s}^*(v)) \in \mathbf{V}(=)$;

(iii) $\mathbf{M} \models Euv$ [s] iff $\mathbf{s}^*(u) < \mathbf{s}^*(v)$;

(iv) $\mathbf{M} \models \sim A$ [s] iff $\mathbf{M} \not\models A$ [s];

(v) $\mathbf{M} \models A \vee B$ [s] iff $\mathbf{M} \models A$ [s] or $\mathbf{M} \models B$ [s];

(vi) $\mathbf{M} \models \Lambda v A$ [s] iff $\mathbf{M} \models A$ [(d/v)s], for all $d \in \mathbf{D}$, if v is an individual variable;

(vii) $\mathbf{M} \models \Lambda v A$ [s] iff $\mathbf{M} \models A$ [(t/v)s], for all $t \in \mathbf{T}$, if v is a moment variable;

(viii) $\mathbf{M} \models \Pi^{\alpha} A$ [s] iff $\mathbf{M} \models A$ [$(\mathbf{p}/\alpha)$s];

(ix) $\mathbf{M} \models \Delta^{\alpha} \mathrm{A}$ [s] iff $\mathbf{M} \models A$ [$(\mathbf{n}/\alpha)$s]; and

(x) $\mathbf{M} \models \Gamma^{\alpha} \mathrm{A}$ [s] iff $\mathbf{M} \models A$ [$(\mathbf{f}/\alpha)$s].

A formula A is said to be *true in an interpretation* $\mathbf{M}$ if every member of $\mathbf{Z}$ satisfies A under $\mathbf{M}$, which we express as $\mathbf{M} \models A$, and A is *false in an interpretation* $\mathbf{M}$, expressed as $\mathbf{M} \not\models A$, if no member of $\mathbf{Z}$ satisfies A under $\mathbf{M}$. *Validity* is defined as truth in every interpretation, and the notation $\models A$ is used to express that A is valid. A is *satisfiable* if there is an interpretation for which A is satisfied by at least one member of $\mathbf{Z}$. A formula A *logically implies* a formula B if, under every interpretation, any member of $\mathbf{Z}$ satisfying A also satisfies B, and A is *logically equivalent* to B if both A logically implies B and B logically implies A.

Standard results such as that A is logically equivalent to B if and only if $\models A \equiv B$ are forthcoming and are not listed here for each connective. All contexts are extensional in the sense that the substitutivity of logical equivalents theorem goes through, though the proof is not set out here.

The indexical quantifiers are somewhat unusual, though it is clear from clauses (viii)–(x) of D5 that all occurrences of a moment variable bound by one of them could well be replaced by a constant. However, it suits my polemic to do things in this way and it will aid certain comparisons with other systems later. The logical properties of the indexical quantifiers are very similar, and we shall often state results which apply to all of them by using $\mathbf{W}$ to stand for any one of them and $\mathbf{w}$ for the corresponding element $\mathbf{p}$, $\mathbf{n}$ or $\mathbf{f}$ of $\mathbf{T}$.

4. Some Results about L*

In this section we draw attention to some features of L* which will be of use in the sequel. First, two technical lemmas.

Lemma 1

If v_i is an individual (moment) variable not free in A, and q is any member of **D** (of **T**), then

$$\mathbf{M} \models A\ [\mathbf{s}] \qquad \text{iff} \qquad \mathbf{M} \models A\ [(q/v_i)\mathbf{s}]$$

Proof: By induction on the length of A.

Case 1 (basis): A is an atomic formula containing just the terms $u_1, \ldots u_n$, and v_i is none of these. Hence the lemma holds trivially in this case.

Case 2: A is $\sim B$.
Suppose $\mathbf{M} \models \sim B\ [\mathbf{s}]$
Now, $\mathbf{M} \models B\ [\mathbf{s}]$ iff $\mathbf{M} \models B\ [(q/v_i)\mathbf{s}]$, by induction hypothesis
Hence $\mathbf{M} \not\models B\ [(q/v_i)\mathbf{s}]$
Hence $\mathbf{M} \models \sim B\ [(q/v_i)\mathbf{s}]$.

Case 3: A is $B \vee C$. By the induction hypothesis, if **U** is the appropriate one of **D** and **T**, then for any $q \in \mathbf{U}$,
$\mathbf{M} \models B\ [\mathbf{s}]$ iff $\mathbf{M} \models B\ [(q/v_i)\mathbf{s}]$
and $\mathbf{M} \models C\ [\mathbf{s}]$ iff $\mathbf{M} \models C\ [(q/v_i)\mathbf{s}]$
Hence $\mathbf{M} \models B\ [\mathbf{s}]$ or $\mathbf{M} \models C\ [\mathbf{s}]$ iff $\mathbf{M} \models B\ [(q/v_i)\mathbf{s}]$ or $\mathbf{M} \models C\ [(q/v_i)\mathbf{s}]$
Hence $\mathbf{M} \models B \vee C\ [\mathbf{s}]$ iff $\mathbf{M} \models B \vee C\ [(q/v_i)\mathbf{s}]$

Case 4: A is $\Lambda v_j B$.
(a) Suppose v_i is v_j. Then
$\mathbf{M} \models \Lambda v_j B\ [\mathbf{s}]$
iff $\mathbf{M} \models B\ [(q'/v_j)\mathbf{s}]$, for all $q' \in \mathbf{U}$
iff $\mathbf{M} \models B\ [(q'/v_j)(q/v_i)\mathbf{s}]$, since
$(q'/v_j)(q/v_i)\mathbf{s} = (q'/v_j)\mathbf{s}$, for all $q, q' \in \mathbf{U}$.
iff $\mathbf{M} \models \Lambda v_j B\ [(q/v_j)\mathbf{s}]$, for all $q \in \mathbf{U}$.
(b) Suppose v_i is not v_j. Then
$\mathbf{M} \models \Lambda v_j B\ [\mathbf{s}]$
iff $\mathbf{M} \models B\ [(q'/v_j)\mathbf{s}]$, for all $q' \in \mathbf{U}$
iff $\mathbf{M} \models B\ [(q/v_i)(q'/v_j)\mathbf{s}]$, for all $q, q' \in \mathbf{U}$, by induction hypothesis
iff $\mathbf{M} \models B\ [(q'/v_j)(q/v_i)\mathbf{s}]$, for all $q, q' \in \mathbf{U}$, since the replacement operations on the sequence s commute under the hypothesis of this subcase
iff $\mathbf{M} \models \Lambda v_j B\ [(q/v_i)\mathbf{s}]$, for all $q \in \mathbf{U}$.

Case 5: A is $\mathbf{W}^{\alpha}B$, where $\mathbf{W}$ is Π, Δ or Γ and $\mathfrak{w}$ the corresponding element $\mathbf{p}$, $\mathbf{n}$ or $\mathbf{f}$ of $\mathbf{T}$. The proof here is similar to that in case 4:

(a) Suppose v_i is α. Then

$\mathbf{M} \models \mathbf{W}^{\alpha}B$ [s]

iff $\mathbf{M} \models B$ [(w/α)s]

iff $\mathbf{M} \models B$ [(w/α)(q/v_i)s], for any $q \epsilon \mathbf{T}$

iff $\mathbf{M} \models \mathbf{W}^{\alpha}B$ [(q/v_i)s], for any $q \epsilon \mathbf{T}$

(b) Suppose v_i is not α. Then

$\mathbf{M} \models \mathbf{W}^{\alpha}B$ [s]

iff $\mathbf{M} \models B$ [(w/α)s]

iff $\mathbf{M} \models B$ [(q/v_i)(w/α)s], for all $q \epsilon \mathbf{U}$

iff $\mathbf{M} \models B$ [(w/α)(q/v_i)s], for all $q \epsilon \mathbf{U}$

iff $\mathbf{M} \models \mathbf{W}^{\alpha}B$ [(q/v_i)s], for all $q \epsilon \mathbf{U}$.

Lemma 2

If v_i is an individual or a moment variable not free in A, and v_j is free for v_i in A, then

$\mathbf{M} \models A$ [s] iff $\bar{\mathbf{M}} \models (v_i/v_j)A$ [(s*(v_j)/v_i)s].

Proof. By induction on the length of A.

Case 1 (basis): A is an atomic formula containing just the terms $u_1, \ldots, u_n$, and v_i is not any of these.

There are two possibilities:

(a) v_j is not one of $u_1, \ldots, u_n$, in which case $(v_i/v_j)A$ is A and the proof is trivial.

(b) v_j is one of $u_1, \ldots, u_n$, say u_k. Then

$\mathbf{M} \models A(u_1, \ldots, u_n)$ [s]

iff (s*(u_1), . . . ,s*(u_{k-1}),((s*(u_k)/v_i)s)*(v_i),s*(u_{k+1}), . . . ,s*(u_n))ϵV(A)

iff $\mathbf{M} \models A(u_1, \ldots, v_i, \ldots, u_n)$ [(s*(u_k)/v_i)s].

Case 2: A is $\sim B$. Then $(v_i/v_j)A$ is $\sim(v_i/v_j)B$. By induction hypothesis,

$\mathbf{M} \models B$ [s] iff $\mathbf{M} \models (v_i/v_j)B$ [(s*(v_j)/v_i)s]

iff $\mathbf{M} \not\models B$ [s] iff $\mathbf{M} \not\models (v_i/v_j)B$ [(s*(v_j)/v_i)s]

iff $\mathbf{M} \models \sim B$ [s] iff $\mathbf{M} \models \sim(v_i/v_j)B$ [s*(v_j)/v_i)s]

Case 3: A is $B \vee C$. Then $(v_i/v_j)A$ is $(v_i/v_j)B \vee (v_i/v_j)C$.

By induction hypothesis,

$\mathbf{M} \models B$ [s] iff $\mathbf{M} \models (v_i/v_j)B$ [(s*(v_j)/v_i)s]

and $\mathbf{M} \models C$ [s] iff $\mathbf{M} \models (v_i/v_j)C$ [s*(v_j)/v_i)s].

Hence, $\mathbf{M} \models B$ [s] or $\mathbf{M} \models C$ [s] iff

$\mathbf{M} \models (v_i/v_j)B$ [(s*(v_j)/v_i)s] or $\mathbf{M} \models (v_i/v_j)C$ [(s*(v_j)/v_i)s]

Hence, $\mathbf{M} \models B \vee C$ [s] iff $\mathbf{M} \models (v_i/v_j)B \vee (v_i/v_j)C$ [(s*(v_j)/v_i)s].

Case 4: A is $\Lambda v_k B$.

(a) v_k is neither v_i nor v_j. Then $(v_i/v_j)A$ is $\Lambda v_k(v_i/v_j)B$ and
$\mathbf{M} \models \Lambda v_k B\ [\mathbf{s}]$
iff $\mathbf{M} \models B\ [(q/v_k)\mathbf{s}]$, for every $q \epsilon \mathbf{U}$, where $\mathbf{U}$ is appropriate one of $\mathbf{D}$ and $\mathbf{T}$
iff $\mathbf{M} \models (v_i/v_j)B\ [(((q/v_k)\mathbf{s})^*(v_j)/v_i)(q/v_k)\mathbf{s}]$, for every $q \epsilon \mathbf{U}$, by induction hypothesis
iff $\mathbf{M} \models (v_i/v_j)B\ [(q/v_k)(\mathbf{s}^*(v_j)/v_i)\mathbf{s}]$, for every $q \epsilon \mathbf{U}$, since $(((q/v_k)\mathbf{s})^*(v_j)/v_i)(q/v_k)\mathbf{s} = (\mathbf{s}^*(v_j)/v_i)(q/v_k)\mathbf{s} = (q/v_k)(\mathbf{s}^*(v_j)/v_i)\mathbf{s}$
iff $\mathbf{M} \models \Lambda v_k(v_i/v_j)B\ [(\mathbf{s}^*(v_j)/v_i)\mathbf{s}]$.

(b) v_k is v_i. Then v_j is not free for v_i in A.

(c) v_k is v_j. Then $(v_i/v_j)A$ is $\Lambda v_i(v_i/v_k)B$ and $\mathbf{M} \models \Lambda v_k B\ [\mathbf{s}]$
iff $\mathbf{M} \models B\ [(q/v_k)\mathbf{s}]$, for every $q \epsilon \mathbf{U}$
iff $\mathbf{M} \models (v_i/v_k)B\ [(((q/v_k)\mathbf{s})^*(v_k)/v_i)\mathbf{s}]$, for all $q \epsilon \mathbf{U}$ by the induction hypothesis
iff $\mathbf{M} \models (v_i/v_k)B\ [(q/v_i)\mathbf{s}]$, since $((q/v_k)\mathbf{s})^*(v_k) = q$
iff $\mathbf{M} \models \Lambda v_i(v_i/v_k)B\ [(\mathbf{s}^*(v_k)/v_i)\mathbf{s}]$, by lemma 1.

Case 5: A is $\mathbf{W}^\alpha B$.

(a) α is neither v_i nor v_j. Then $(v_i/v_j)A$ is $\mathbf{W}^\alpha(v_i/v_j)B$ and the proof here proceeds as in case 4(a).

(b) α is v_i. Then v_j is not free for v_i.

(c) α is v_j. Then $(v_i/v_j)A$ is $\mathbf{W}^{v_i}(v_i/\alpha)B$ and the proof here is like that in case 4(c).

Corollary 2.1
If v_j is not free in A but is free for v_i in A, then A is logically equivalent to $(v_i/v_j)A$.

Proof. For any interpretation $\mathbf{M}$ and sequence $\mathbf{s} \epsilon \mathbf{Z}$, $\mathbf{M} \models A\ [\mathbf{s}]$
iff $\mathbf{M} \models (v_i/v_j)A\ [(\mathbf{s}^*(v_j)/v_i)\mathbf{s}]$, by lemma 2
iff $\mathbf{M} \models (v_i/v_j)A\ [(\mathbf{s}^*(v_i)/v_i)(\mathbf{s}^*(v_j)/v_i)\mathbf{s}]$, by lemma 1
iff $\mathbf{M} \models (v_i/v_j)A\ [\mathbf{s}]$.

The indexical quantifiers behaved very much like the universal quantifier in the proofs of these two lemmas, perhaps justifying our calling them quantifiers. Some difference will become apparent in the following lemmas and theorems.

Lemma 3
$\mathbf{W}^\alpha \sim A$ is logically equivalent to $\sim \mathbf{W}^\alpha A$.

Proof. For any interpretation $\mathbf{M}$ and sequence $\mathbf{s} \epsilon \mathbf{Z}$ $\mathbf{M} \models \mathbf{W}^\alpha \sim A\ [\mathbf{s}]$
iff $\mathbf{M} \models \sim A\ [(\mathbf{w}/\alpha)\mathbf{s}]$
iff $\mathbf{M} \not\models A\ [(\mathbf{w}/\alpha)\mathbf{s}]$
iff $\mathbf{M} \not\models \mathbf{W}^\alpha A\ [\mathbf{s}]$
iff $\mathbf{M} \models \sim \mathbf{W}^\alpha A\ [\mathbf{s}]$
Hence, $\models \mathbf{W}^\alpha \sim A \equiv \sim \mathbf{W}^\alpha A$.

Corollary 3.1
$\sim \mathbf{W}^{\alpha} \sim A$ is logically equivalent to $\mathbf{W}^{\alpha} A$.

Lemma 4
If γ is not free in A or B, α is free for γ in A and β is free for γ in B, then $\mathbf{W}^{\alpha} A \vee \mathbf{W}^{\alpha} B$ is logically equivalent to $\mathbf{W}^{\gamma}((\gamma/\alpha)A \vee (\gamma/\beta)B)$.

Proof. For any interpretation $\mathbf{M}$ and $\mathbf{s} \in \mathbf{Z}$,

$\mathbf{M} \models \mathbf{W}^{\alpha} A \vee \mathbf{W}^{\beta} B$ [s]
iff $\mathbf{M} \models \mathbf{W}^{\alpha} A$ [s] or $\mathbf{M} \models \mathbf{W}^{\beta} B$ [s]
iff $\mathbf{M} \models \mathbf{W}^{\gamma}(\gamma/\alpha)A$ [$(\mathbf{s}^*(\alpha)/\gamma)\mathbf{s}$] or $\mathbf{M} \models \mathbf{W}^{\gamma}(\gamma/\beta)B$ [$(\mathbf{s}^*(\beta)/\gamma)\mathbf{s}$], by lemma 2
iff $\mathbf{M} \models (\gamma/\alpha)A$ [$(\mathbf{w}/\gamma)(\mathbf{s}^*(\alpha)/\gamma)\mathbf{s}$] or $\mathbf{M} \models (\gamma/\beta)B$ [$(\mathbf{w}/\gamma)(\mathbf{s}^*(\beta)/\gamma)\mathbf{s}$]
iff $\mathbf{M} \models (\gamma/\alpha)A \vee (\gamma/\beta)B$ [$(\mathbf{w}/\gamma)\mathbf{s}$], since $(\mathbf{w}/\gamma)(\mathbf{s}^*(\alpha)/\gamma)\mathbf{s} = (\mathbf{w}/\gamma)(\mathbf{s}^*(\beta)/\gamma)\mathbf{s} = (\mathbf{w}/\gamma)\mathbf{s}$
iff $\mathbf{M} \models \mathbf{W}^{\gamma}((\gamma/\alpha)A \vee (\gamma/\beta)B)$ [s]

Corollary 4.1
If γ is not free in A or B, α is free for γ in A and β is free for γ in B, then the following are pairs of logically equivalent formulae:

$\mathbf{W}^{\alpha} A \,\&\, \mathbf{W}^{\beta} B$ and $\mathbf{W}^{\gamma}((\gamma/\alpha)A \,\&\, (\gamma/\beta)B)$
$\mathbf{W}^{\alpha} A \supset \mathbf{W}^{\beta} B$ and $\mathbf{W}^{\gamma}((\gamma/\alpha)A \supset (\gamma/\beta)B)$
$\mathbf{W}^{\alpha} A \equiv \mathbf{W}^{\beta} B$ and $\mathbf{W}^{\gamma}((\gamma/\alpha)A \equiv (\gamma/\beta)B)$.

Theorem 5
If A contains the subformulae $\mathbf{W}^{\alpha_1} B_1, \ldots, \mathbf{W}^{\alpha_n} B_n$, then A is logically equivalent to $\mathbf{W}^{\beta}(\beta/\alpha_1, \ldots, \alpha_n)A$, provided β is not free in A and $\alpha_1, \ldots, \alpha_n$ are each free for β in A.

Proof. By induction on the length of A.

Case 1 (basis): A is $\mathbf{W}^{\alpha} B$, which is logically equivalent to $\mathbf{W}^{\beta}(\beta/\alpha)B$ by corollary 2.1.

Case 2: A is $\sim C(\mathbf{W}^{\alpha_1} B_1, \ldots, \mathbf{W}^{\alpha_n} B_n)$. Then

$\mathbf{M} \models \sim C(\mathbf{W}^{\alpha_1} B_1, \ldots, \mathbf{W}^{\alpha_n} B_n)$ [s]
iff $\mathbf{M} \not\models C(\mathbf{W}^{\alpha_1} B_1, \ldots, \mathbf{W}^{\alpha_n} B_n)$ [s]
iff $\mathbf{M} \not\models \mathbf{W}^{\beta}(\beta/\alpha_1, \ldots, \alpha_n)C$ [s], by induction hypothesis
iff $\mathbf{M} \models \sim \mathbf{W}^{\beta}(\beta/\alpha_1, \ldots, \alpha_n)C$ [s]
iff $\mathbf{M} \models \mathbf{W}^{\beta} \sim (\beta/\alpha_1, \ldots, \alpha_n)C$ [s], by lemma 3.

Case 3:

(a) A is $C(\mathbf{W}^{\alpha_1} B_1, \ldots, \mathbf{W}^{\alpha_k} B_k) \vee D(\mathbf{W}^{\gamma_1} A_1, \ldots, \mathbf{W}^{\gamma_m} A_m)$. Then

$\mathbf{M} \models C(\mathbf{W}^{\alpha_1} B_1, \ldots, \mathbf{W}^{\alpha_k} B_k) \vee D(\mathbf{W}^{\gamma_1} A_1, \ldots, \mathbf{W}^{\gamma_m} A_m)$ [s]
iff $\mathbf{M} \models C(\mathbf{W}^{\alpha_1} B_1, \ldots, \mathbf{W}^{\alpha_k} B_k)$ [s] or $\mathbf{M} \models D(\mathbf{W}^{\gamma_1} A_1, \ldots, \mathbf{W}^{\gamma_m} A_m)$ [s]
iff $\mathbf{M} \models \mathbf{W}^{\beta}(\beta/\alpha_1, \ldots, \alpha_k)C$ [s] or $\mathbf{M} \models \mathbf{W}^{\beta}(\beta/\gamma_1, \ldots, \gamma_m)D$ [s],
by induction hypothesis

iff $\mathbf{M} \models \mathbf{W}^{\beta}((\beta/\alpha_1, \ldots, \alpha_k)C \vee (\beta/\gamma_1, \ldots, \gamma_m)D)$ [s], by lemma 4.

(b) A is $C \vee D$, but one disjunct, say D, contains no subformulae of the kind $\mathbf{W}^{\alpha}B$. Then

$\mathbf{M} \models C(\mathbf{W}^{\alpha_1}B_1, \ldots, \mathbf{W}^{\alpha_n}B_n) \vee D$ [s]

iff $\mathbf{M} \models \mathbf{W}^{\beta}(\beta/\alpha_1, \ldots, \alpha_n)C$ [s] or $\mathbf{M} \models D$ [s]

iff $\mathbf{M} \models (\beta/\alpha_1, \ldots, \alpha_n)C$ [(w/β)s] or $\mathbf{M} \models D$ [(w/β)s], by lemma 1

iff $\mathbf{M} \models \mathbf{W}^{\beta}((\beta/\alpha_1, \ldots, \alpha_n)C \vee D)$ [s]

Case 4: A is $\Lambda\nu C(\mathbf{W}^{\alpha_1}B_1, \ldots, \mathbf{W}^{\alpha_n}B_n)$, and according to the formation rules ν cannot be one of $\alpha_1, \ldots, \alpha_n$. Hence,

$\mathbf{M} \models \Lambda\nu C(\mathbf{W}^{\alpha_1}B_1, \ldots, \mathbf{W}^{\alpha_n}B_n)$ [s]

iff $\mathbf{M} \models C(\mathbf{W}^{\tilde{\alpha}_1}B_1, \ldots, \mathbf{W}^{\tilde{\alpha}_n}B_n)$ $[(q/\nu)\mathrm{s}]$, for every $q \epsilon \mathbf{U}$ which is **D** or **T**

iff $\mathbf{M} \models \mathbf{W}^{\beta}(\beta/\alpha_1, \ldots, \alpha_n)C$ $[q/\nu)\mathrm{s}]$, for every $q \epsilon \mathbf{U}$

iff $\mathbf{M} \models (\beta/\alpha_1, \ldots, \alpha_n)C$ $[(\mathrm{w}/\beta)(q/\nu)\mathrm{s}]$, for every $q \epsilon \mathbf{U}$

iff $\mathbf{M} \models (\beta/\alpha_1, \ldots, \alpha_n)C$ $[(q/\nu)(\mathrm{w}/\beta)\mathrm{s}]$, for every $q \epsilon \mathbf{U}$

iff $\mathbf{M} \models \mathbf{W}^{\beta}\Lambda\nu(\beta/\alpha_1, \ldots, \alpha_n)C$ [s].

Case 5: A is $\mathbf{O}^{\gamma}C(\mathbf{W}^{\alpha_1}B_1, \ldots, \mathbf{W}^{\alpha_n}B_n)$, where **O** is Π, Δ or Γ. The proof here is similar to case 4.

The fundamental principle of our analysis of tenses, that the point of view of the speaker dominates all subordinate clauses, can be roughly expressed by saying that the point of view from which time is divided into past, present and future is the same in all clauses. The indexical quantifiers serve in the system presented here as indicators of past, present and future, so that our principle is nicely reflected and perhaps more precisely expressed in theorem 5.

Chapter IV

Formulation of Tensed Statements

A distinction between simple and complex tenses is outlined in the first section, together with some indication of a classification of tenses as past, present and future. Attention is focused on statements containing more than one tense in the following section, and the so-called sequence of tenses rules are discussed there. Later on, in the last section, the distinctions outlined here in these two sections are made more precise in the form of definitions within the framework of our formal system. But before that, the language L* is generalised somewhat into our fully-fledged language L. This language will be compared with some of the other systems of tense logic that have been proposed in Chapter V.

1. Simple and Complex Tenses

There are three obvious kinds of formulae within L* which can be used to express tensed statements, namely formulae of the kinds $\Pi^{\alpha}A$, $\Delta^{\alpha}A$ and $\Gamma^{\alpha}A$. Where A contains no occurrences of any moment variables, free or bound, apart from α, the tenses expressed by these formulae will be said to be simple tenses. Moreover, the tenses expressed by such formulae will be said to be, respectively, past, present and future tenses. The simple past, present and future are intended to correspond to those special cases of Reichenbach's analysis where the point of reference and the point of the event coincide, so that what is here called the simple future differs from what Reichenbach called the simple future. Why Reichenbach opted for this lack of symmetry in the underlying structure of tenses I do not really know—perhaps he was guided by grammatical considerations. This suggests the need for some indication of the relation between grammatical and logical forms of tenses if our definitions are to characterise tenses in English, though I see no immediate hope of formalising this relation.

Certain elementary ways of reading the indexical quantifiers in their occurrences in formulae spring to mind. Suppose P_1^1 is a 1-place predicate letter standing for 'is red', which we shall abbreviate to P. Since the evaluation of the formula $\Delta^{\alpha\alpha}Pa$ assigns **n**—the present moment—to α, the formula might be read 'a is red now', 'At present, a is red', or simply 'a is red', where it is understood that the verb is not to be taken as tenseless. Similarly, $\Pi^{\alpha\alpha}Pa$ can be read as 'a was red' and $\Gamma^{\alpha\alpha}Pa$ as 'a will be red'. 'It will be the case that a is red' is not such a perspicuous reading of the latter formula since it suggests that Γ is, like Prior's F,

a propositional operator for which iteration is possible, rather than forming such an intimate part of the tensed predicate 'will be red'. These "is, was or will be the case that" constructions strike me as relatively unusual English, although it must be admitted that they do occasionally occur. Rather than paraphrase ordinary tensed predicates into this propositional operator form, I shall adopt the policy of trying to paraphrase statements like 'It is (was, will be) the case that John is bald' in such a way that it becomes more apparent that the tense belongs to the structure of the predicate, so we have in this case 'John is (was, will be) bald'.

The relative merits of these opposing views are considered in some detail in the next chapter; but to bring out the difference a little more now, consider two examples of Prior's that have already been mentioned in previous chapters:

(1) He said he was ill.
(2) He said he would be in London now.

I do not want to get involved here in the problems of analysis of *oratio obliqua* statements, but it seems hardly contentious to assume simply that 'He says that' can be represented as a relativised propositional operator by analogy with Hintikka's knowledge and belief operators.[1] Since I am particularly interested in the tenses expressed by (1) and (2), however, some allowance must be made for modification by tenses, and accordingly '*b* says that *A*' will be represented as ${}^{\alpha}S_{b}{}^{\alpha}A$. With this notation, (1) can be expressed as $\Pi^{\alpha\alpha}S_{u}{}^{\alpha}Au$, and the point I want to emphasise here is that there is no need, according to this formulation, to paraphrase (1) as 'He said that he is ill' as Prior recommends; from our point of view, (1) is quite all right as it stands. (2) arose in connection with the discussion of the non-eliminability of 'now', and will serve as our first substantial illustration of the expressive power of the indexical quantifiers. For the problems that (2) presents to Prior do not arise since it can be formulated as $\Delta^{\alpha}\Pi^{\beta\beta}S_{a}{}^{\alpha}Aa$.

Although Δ is intended to enable us to formalise occurrences of 'now' in English, it is not simply a " 'now'-operator" in the sense that this is its only function. In general, Δ is used to formalise the present tense, and what seemed to Prior the unusual behaviour of 'now' is from my point of view reflected in a uniform way as just one aspect of the present tense. So Δ^{α} is more correctly called the present tense operator. Similarly, Π^{α} is called the past tense operator and Γ^{α} the future tense operator. 'Now' and 'then' usually indicate cross-references to moments, just like pronouns are used to indicate cross reference to individuals, and are not in general taken to be essential components of the readings of formulae containing indexical quantifiers.

This gives some idea of the simple tenses and how they are divided into past, present and future tenses. Now I want to outline complex tenses and indicate how they are divided into past, present and future tenses. Let us begin with what Reichenbach calls the past perfect, usually expressed by the 'had -ed' form. We

[1] Hintikka, *Knowledge and Belief* (New York, 1962).

said that this was to be analysed in terms of an existential quantification over moments earlier than the reference point in the past, and accordingly a statement like 'John had gone' can be expressed in the form $\Pi^{\alpha} \mathrm{V}\beta(E\beta\alpha \;\&\; {}^{\beta}Aa)$. It will be convenient to have a less clumsy formulation which we achieve with the following definition:

$$\mathrm{Df}P \qquad {}^{\alpha}P^{\beta}\,{}^{\beta}A = \mathrm{df}\ \mathrm{V}\beta(E\beta\alpha \;\&\; {}^{\beta}A)$$

Every occurrence of β in A is thus bound by an existential quantification over moments earlier than α in wffs of the kind ${}^{\alpha}P^{\beta}{}^{\beta}A$, where α remains free, and the formula in question now becomes $\Pi^{\alpha\alpha}P^{\beta\beta}Aa$. This suggests that P be correlated with the word 'before' so that the former formula can be read '*a As* before α' and a formula lika ${}^{\alpha}P^{\beta}({}^{\beta}Aa \;\&\; {}^{\alpha}Bb)$ might be read '*a As* before *b Bs*'.

There is a similar construction for 'after'. With the definition

$$\mathrm{Df}F \qquad {}^{\alpha}F^{\beta\beta}A = \mathrm{df}\ \mathrm{V}\beta(E\alpha\beta \;\&\; {}^{\beta}A)$$

we can express 'John would come if Jane met him on the way here', for example, in the form $\Pi^{\alpha}({}^{\alpha}Aab \supset {}^{\alpha}F^{\beta\beta}Ba)$. These definitions obviously parallel the usual semantic interpretations of Prior's P and F operators, and in the same vein we can introduce the following definitions:

$$\mathrm{Df}H \qquad {}^{\alpha}H^{\beta\beta}A = \mathrm{df}\ \Lambda\beta(E\beta\alpha \supset {}^{\beta}A)$$
$$\mathrm{Df}G \qquad {}^{\alpha}G^{\beta\beta}A = \mathrm{df}\ \Lambda\beta(E\alpha\beta \supset {}^{\beta}A)$$

The following results are readily forthcoming:

Lemma 6

(i) $\mathbf{M} \models {}^{\alpha}G^{\beta}A$ [s] iff $\mathbf{M} \models A$ [(t/β)s], for every $t \epsilon \mathbf{T}$ s.t. $\mathrm{s}^*(\alpha) < t$;
(ii) $\mathbf{M} \models {}^{\alpha}F^{\beta}A$ [s] iff $\mathbf{M} \models A$ [(t/β)s], for some $t \epsilon \mathbf{T}$ s.t. $\mathrm{s}^*(\alpha) < t$;
(iii) $\mathbf{M} \models {}^{\alpha}H^{\beta}A$ [s] iff $\mathbf{M} \models A$ [(t/β)s], for every $t \epsilon \mathbf{T}$ s.t. $t < \mathrm{s}^*(\alpha)$; and
(iv) $\mathbf{M} \models {}^{\alpha}P^{\beta}A$ [s] iff $\mathbf{M} \models A$ [(t/β)s], for some $t \epsilon \mathbf{T}$ s.t. $t < \mathrm{s}^*(\alpha)$.

Lemma 7

(i) $\models \sim {}^{\alpha}F^{\beta}\sim A \equiv {}^{\alpha}G^{\beta}A$
(ii) $\models \sim {}^{\alpha}P^{\beta}\sim A \equiv {}^{\alpha}H^{\beta}A$

Corollary 7.1

The following pairs of formulae are logically equivalent: $\sim {}^{\alpha}F^{\beta}A$ and ${}^{\alpha}G^{\beta}\sim A$, $\sim {}^{\alpha}G^{\beta}A$ and ${}^{\alpha}F^{\beta}\sim A$, $\sim {}^{\alpha}P^{\beta}A$ and ${}^{\alpha}H^{\beta}\sim A$, and finally $\sim {}^{\alpha}H^{\beta}A$ and ${}^{\alpha}P^{\beta}\sim A$.

As a preliminary indication of the way I want to characterise tenses, certain kinds of more elementary formulae are arranged under appropriate headings in the following table, where the wff A contains just the one free moment variable indicated.

		Past	*Present*	*Future*
Simple		$\Pi^{\alpha\alpha}A$	$\Delta^{\alpha\alpha}A$	$\Gamma^{\alpha\alpha}A$
Complex	*Anterior*	$\Pi^{\alpha\alpha}P^{\beta}A$	$\Delta^{\alpha\alpha}P^{\beta\beta}A$	$\Gamma^{\alpha\alpha}P^{\beta\beta}A$
	Posterior	$\Pi^{\alpha\alpha}F^{\beta\beta}A$	$\Delta^{\alpha\alpha}F^{\beta\beta}A$	$\Gamma^{\alpha\alpha}F^{\beta\beta}A$

I shall conclude this section with some comments on how the characterisation of tenses suggested here corresponds with Diver's scheme. Simple tenses correspond to Diver's definite tenses, and we have seen something of their expression in English earlier in this section and in Chapter II. Diver's notion of indefiniteness is characterised by existential quantification, but there is some modification of Diver's scheme here since all of what we have called complex tenses are characterised as indefinite in this way. Thus, the past anterior, for example, (corresponding to his 'past, before' and usually expressed by the 'had -ed' form), is said to express possibly more than one occurrence of the event in question at some ill-defined time or times (consider 'He had played several games when we arrived'). This extension of the application of the term 'indefinite' arises because the elements 'before' and 'after' occurring in his analysis correspond here not simply with the 'earlier than' and 'later than' relations, but to elements involving existential quantification. As a consequence, Diver's distinction between sequential and non-sequential tenses disappears, at least as a characterisation of tenses distinct from the division between definite and indefinite. Thus, our distinction between simple and complex tenses, whilst not coinciding with the distinctions as Diver drew them, does represent much of what was intended to underlie the distinctions between definite and indefinite, and between sequential and non-sequential tenses.

A formula is said to be past, present or future tensed if it contains a moment variable bound by, respectively, Π, Δ or Γ. This convention has the consequence that some statements may have more than one of a past, a present and a future tense. Such statements are considered in the following section.

Finally, it is possible at this stage to make some suggestions why Diver might have thought that the definite/indefinite distinction does not apply to the future tenses. We have seen that the simple past (cf. Diver's past definite) is fairly well correlated with the '-ed' inflexion, whilst the present anterior (cf. Diver's past indefinite) is usually expressed by the 'have -ed' form; and certainly the simple past is hardly, if ever, expressed by the 'have -ed' form. On the other hand, the distinction between the present posterior and the simple future is much less well marked than this. The simple future is often expressed by the 'will -O' form; for example, 'His father will be home shortly and he will behave a lot better then'. But the simple '-O' form often used to express the simple present is also commonly used to express the simple future; for example, 'If you come to dinner, you will hear the record then', which we express in the form $\Gamma^{\alpha}({}^{\alpha}Aa \supset {}^{\alpha}Bb)$ since it is clear that the occurrence of 'then' in the second clause identifies the time in the future referred to there with the time indicated in the first clause. Furthermore, not only is the simple future commonly expressed by both the '-O' and the 'will -O' forms, the former of

which is also frequently used to express the simple present; the 'will -O' form is also frequently used to express the present anterior. Consider, for example, 'If you leave now you will see some elephants', which we symbolise in the form $\Delta^{\alpha}({}^{\alpha}Aa \supset {}^{\alpha}F^{\beta\beta}Ba)$. The fact that this distinction amongst future tenses is not so clearly marked by verb inflexions as in other cases and that we need to look at a wider context to resolve possible ambiguity, however, seems to me insufficient to warrant a lack of symmetry between past and future tense structures in our analysis, for this is only a more extreme case of what we find with other tenses. Yet this seems to be the only reason Diver has for advocating this lack of symmetry in his analysis.

2. Sequences of Tenses

Reichenbach considered that his analysis gives some insight into what grammarians call the rules for sequences of tenses which govern the way linguistic tenses adjust to one another in compound sentences with several clauses. He begins with the suggestion that these rules be interpreted according to a principle of the *permanence of the reference point,* that, although the events or states of affairs referred to in the different clauses may occur at different times, the reference point remains the same in all clauses. Thus, Reichenbach represents the tenses in the sentence

(3) I had mailed the letter when John came and told me the news

by the diagram

(4) 1st clause: E_1–R_1 — S
2nd clause: R_2, E_2 — S
3rd clause: R_3, E_3 — S

where the three reference points R_1, R_2 and R_3 coincide. With the resources of our systems, (3) can be expressed in the form $\Pi^{\alpha\alpha}P^{\beta\beta}Aa$ & $\Pi^{\gamma\gamma}Bb$ & $\Pi^{\delta\delta}Cba$, which, according to theorem 5, is equivalent (provided γ and δ are free for α in B and C), to

(5) $\Pi^{\alpha}({}^{\alpha}P^{\beta\beta}Aa$ & ${}^{\alpha}Bb$ & ${}^{\alpha}Cba)$.

The principle of the permanence of the reference point, then, can be expressed within the framework of our formal system by requiring that, once the indexical quantifiers have been moved to the front of a wff according to theorem 5, just one remains.

Following Reichenbach, we have formulated the second clause of (3) in the simple past since the verb inflexion '-ed' is usually used to express this tense. Now, it would be grammatically incorrect to say

(6) I had mailed the letter when John has come and told me the news,

and Reichenbach's principle suggests why this should be so. For the tense usually expressed by the verb form occurring in the clause 'John has come' is the present anterior, which would involve a distinct point of reference in its formulation and so conflict with the principle of the permanence of the reference point.

Another example of Reichenbach's illustrating the same point is

(7) John has not decided which train he will take.

First, we need a definite description operator defined thus:

Df *Th* $/A(Th\nu) = \text{df}\ \ V\nu(\Lambda u(Bu \equiv u{=}\nu)\ \&\ A\nu)$,

the oblique stroke indicating the scope of the definite description. For our purposes, the unique quantifier will usually take the smallest possible scope and we shall adopt the convention of dropping the oblique stroke where this is so. (7) can now be formulated thus:

(8) $\sim \Delta^{\alpha\alpha} P^{\beta\beta} AaThx^{\alpha} F^{\gamma\gamma} Bax,$

where *Axy* stands for '*x* decides on *y*' and *Bxy* for '*y* is a train *x* takes'. It would be incorrect to say

(9) John did not decide which train he will take,

where again the explanation is that the incorrect verb form (John did not decide) is usually associated with a tense (the past anterior) which would require another reference point.

Informative as this principle may be in cases like these, however, it is clear that it is not in general true that a sentence is used to express a temporal ordering of events in the neighbourhood of just one point of reference. An example we have already encountered is (2) of the last section. A more perspicuous example for which the resources of L* are adequate is

(10) John will repair the vase that Mrs. Smith broke,

which we express in the form

(11) $\Delta^{\alpha} \Pi^{\beta\alpha} AaThx^{\beta} Bb.$

With one reference point in the past and the other in the future, there can be no question of their being identical. Reichenbach does indeed recognise this point, but there are some aspects of the account he develops which seem to me disputable.

Reichenbach begins this part of his discussion with the suggestion that "when a time determination is added, such as is given by words like 'now' or 'yesterday', or by a non-reflexive symbol like 'November 7 1944', it is referred, not to the event, but to the reference point of the sentence."[2] So we have, for example, 'Smith had

[2] Reichenbach, *Elements of Symbolic Logic*, p. 274. All further quotations from this work in the present section are indicated in the body of the text.

read the report when it was discussed yesterday', and 'The situation had been deteriorating when on November 7 1944, matters began to improve', tending to bear out this contention.

The first part of this claim, involving what Reichenbach called token reflexive expressions, is substantially in agreement with what has already been said about the indexical character of the reference points. To do full justice to expressions like 'yesterday', 'tomorrow', 'ten days ago', and so on, some metrical concepts would have to be introduced so that we could analyse these expressions in terms of some specified duration of time at some specified interval earlier than or later than the present. But this will not be pursued here. Tensed dated statements (as opposed to tenseless dated ones) express that the time indicated by the tense is identical to that indicated by the date. So, 'Bertrand Russell died on 2nd February 1970' is expressed in the form $\Pi^{\alpha}({}^{\alpha}Aa \,\&\, \alpha = m)$ where m stands for the date '2nd February 1970'. Whether dates always specify the reference point in this way, however, is something I doubt—consider 'I had already tried to contact him on the 4th, 5th and 6th of the month before eventually succeeding on the 7th'. This is as we might expect in view of the indexical character attributed here to the reference points: 'now', 'then', 'yesterday', 'three days hence', and so on, are indexical expressions whereas dates do not depend so heavily on the context of utterance for their sense.

In the light of his claim, Reichenbach suggested that we might speak of the *positional use of the reference point* in contexts where a relatively specific time determination appears explicitly. He then goes on to propose that the positional use of the reference point extends to cases where moments in time are "compared by means of words like 'when', 'before', or 'after' ", where "it is the reference points to which the comparison refers directly, not events" (p. 294). An earlier example, 'I had mailed the letter when John came and told me the news', is a case in point: though the events described are not all simultaneous, the points of reference in each clause are, and this is indicated by the word 'when'. Thus, the rule of the permanence of the reference point can be seen as a special case of the positional use of the reference point, indicated in this case by the word 'when'. On the other hand, in the case of

(12) He telephoned before he came,

Reichenbach suggests that there are two reference points compared by the temporal relation of precedence, and since both clauses of (12) are in the past tense, this would require two reference points in the past. Now this consequence would be very awkward from our point of view. It would mark the thin end of the wedge, leading to an unlimited number of reference points, which was Prior's objection to Reichenbach's account, and would make it very difficult to maintain the thesis of the indexical character of the reference points and the principle that the point of view of the speaker dominates all subordinate contexts.

It seems that Reichenbach was lead to this conclusion because the verbs in both clauses of (12) have the same kind of inflexion, and this verb form he correlates

with his simple past structure. For he says (where 'R_1' and 'R_2' are respectively the two reference points of (12)), "R_1 is said to be before R_2; but, at least, the tenses used here have the same structure" (p. 295), and presumably he takes it to be a virtue of his account that the tenses are ascribed the same structure because they have the same linguistic form. However, we have seen in Chapter II and in this chapter that there are good reasons for not strictly correlating the linguistic forms of verbs uniquely with particular semantic structures. In particular, it has been argued that cases in Reichenbach's scheme where the point of reference and the point of the event coincide correspond to Diver's definite tenses, so that Reichenbach's (and my) simple past is Diver's past definite, but that the '-ed' form is by no means always used to express the past definite. Now, since 'before' occurs explicitly in (12), it is natural to suggest in view of Df*P* that (12) be formulated as

(13) $\Pi^{\alpha}({}^{\alpha}P'^{\beta}{}_{\beta}Ba \;\&\; {}^{\alpha}Aa)$,

where *Ba* stands for 'he telephones' and *Aa* for 'he comes'. The plausibility of this analysis is borne out by the fact that a natural reading of (13) in English is

(14) He had telephoned when he came,

since the 'had -ed' form is usually used to express the past anterior tense, and (14) is quite a good paraphrase of (12). So we can retain our analysis which allows just one point of reference in the past, and (12) can be seen as a candidate for the application of the principle of the permanence of the reference point.

Rather than maintaining with Reichenbach that 'before' and 'after' are always used to compare reference points, a more plausible suggestion would be that when used as parts of linguistic tense constructions they serve ultimately to relate moments in time to reference points so that a temporal ordering is established in the neighbourhood of a reference point. This point is reflected within the resources of our formal system since expressions like ${}^{\alpha}P^{\beta}$ and ${}^{\alpha}F^{\beta}$ bind subsequent occurrences of β, but α is free unless bound by an indexical quantifier. And we say 'before' and 'after' serve *ultimately* to relate moments to reference points since it is possible to form iterated sequences like ${}^{\alpha}P^{\beta_1}{}_{\beta_1}P^{\beta_2}\ldots{}^{\beta_n}P^{\gamma}$, but the initial moment variable remains free in such sequences, available for binding by an indexical quantifier.

Sometimes two or three reference points are compared ((2) is a case in point), but my thesis is that at most only the three reference points so far introduced are required. Reichenbach has a particularly interesting example of such a comparison:

(15) He was healthier when I saw him than he is now,

for a problem arises of how to formulate the two-place predicate 'is healthier than' in a tensed context. This brings me to the topic of the next section.

3. Comparisons

Rather than deal immediately with the relation 'is healthier than', let us consider a somewhat more straightforward example to illustrate the problem, namely

(16) I am taller than I was.

Here my present height is compared with my former height, so that the relational predicate 'is taller than' does not simply hold of me now, nor at some former time; it would be more accurate to say that it is true of me relative to the pair comprising these two times. Now, it seems to be something of a well-entrenched attitude, certainly in the case of the Priorean analysis of temporal reference, that something or other can be said to hold of an individual or set of individuals at a single time or otherwise can be analysed into properties of individuals which do hold at a single time. Consequently, statements like (15) and (16) present at least a problem of analysis for those who hold some such view. I shall maintain that in general predicates which hold of individuals relative to two or more times are not, in the final analysis, eliminable in favour of predicates which hold relative to a single time, and this will lead to the development of the fully-fledged language L.

One possibility for the analysis of (16) that springs to mind is to introduce into our formal system real numbers, together with appropriate operations and relations defined on them, and functional terms or measures ascribing numbers to individuals at different times. Thus, h could be a binary function interpreted so that ${}^{\alpha}hx$ is understood as the height of x at α, and $>$ the binary relation 'is greater than' on the real numbers, so that (16) could be expressed in the form

(17) $\Delta^{\alpha}\Pi^{\beta}({}^{\alpha}ha > {}^{\beta}ha)$,

for example (or perhaps $\Delta^{\alpha}\alpha P\beta({}^{\alpha}ha > {}^{\beta}ha)$). In this way the predicate occurring in (16) is reduced to an expression holding of an individual at a single time.

Now, a great deal of plausibility attaches to the possibility of articulating, in terms of measures, (16) and many similar examples which can be constructed by replacing 'taller than' with 'younger than', 'heavier than', 'fatter than', and so on; and perhaps 'is stronger than' and 'is more intelligent than'. But there are two lines of argument against this procedure as an elimination of predicates holding relative to more than one time, the first suggesting a limitation in its generality, and the second, more substantial objection arising from a possible counter argument against the first objection.

Though plausible for the predicates mentioned above, it might be argued that evaluation in terms of numerical measures is distinctly less plausible in the cases of predicates like 'is wiser than', 'is more mature than', 'is as beautiful as', 'is more broad-minded than', 'is more reasonable than', 'is as profoundly moved as', and so on, and we might include 'is healthier than' in this category. For it might be objected that these relations are not connected, so that not all pairs of individuals in the field of such relations are comparable: it might not be possible to say that

some individual is more, less or as wise as some other, or is more, less or as mature as another. Thus, relations such as these do not form part of a set of linear order relations. But, as Ellis[3] proposes, it is at least a necessary condition for a property to be regarded as a quantity, and therefore measurable, that a corresponding set of relations exists which forms a set of linear order relationships.

Perhaps it might be suggested that the account be moderated so that the functions do not necessarily assign numbers, but some other kinds of entities like "the beauty of Jean" which might form incomparable subsets relative to the relation in question. But I reject this for much the same reason that Ellis[4] suggests it misconceived to ask what feature of a system is its mechanical energy or entropy: mechanical energy or entropy are better understood on a comparative basis in terms of the corresponding relations which make measurement possible. In the case of the relations in question, even though 'is more profoundly moved than', 'is less profoundly moved than' and 'is as profoundly moved as', for example, do not form a set of linear order relations, it strikes me as more perspicuous to retain the relational predicates than analyse, say, 'I am more profoundly moved by the music than ever before' along the lines 'The present profundity of my emotion evoked by the music exceeds all previous such profundities of my emotion'.

Now, it might be at this point that the difficulties suggested here are really only epistemological problems of ascertaining a scale of measurement, either because of the vagueness of the comparative quantity in question, or because the quantity in question is better thought of as a cluster of comparative quantities (beauty is a good example) and the procedure of combining assessments on each of these counts is not well-defined. Something like this seems to me the correct view, so that the first line of argument is not successful in showing that in principle no measurement is possible in the case of examples like those mentioned. But now I would like to point out, simply by appealing to standard accounts of measurement, that the introduction of a measure on a set does not amount to the elimination of the comparative relations defined on that set. This was already becoming apparent from the references to a set of linear order relations for a measurable quantity, but to make our ideas somewhat more precise, I shall allude to Kanger's article on measurement.[5] Without spelling out all the details, a measure m for a set **D** assigns a real number to each element of **D**. An aspect of **D** (to be identified with a quantity, say height, loudness or health, for example, under certain conditions) is a structure $\mathbf{Asp} = (\mathbf{U},\mathbf{R}_1,\ldots,\mathbf{R}_k)$, where **U** is a subset of **D** and $\mathbf{R}_1,\ldots,\mathbf{R}_k$ are relations defined on **U**, whose type is $(n_1,\ldots,n_k)$, where $\mathbf{R}_j$ is an n_j-place relation ($j = 1,2,\ldots,k$). A scale is a numerical structure; for example, **S** is an ordinal scale if $\mathbf{S} = (\mathbf{R},=,>,\ldots)$, where **R** is a set of real numbers, $=$ and $>$ the identity and 'greater than' relations on **R**. Then a measurement is said to be a system $(\mathbf{D},\mathbf{Asp},\mathbf{S},m)$ where (i) **Asp** is an aspect and **S** a scale of the same type as

[3] Brian Ellis, *Basic Concepts of Measurement* (Cambridge, 1968), Chapter 2, section A.

[4] Loc. cit.

[5] Stig Kanger, 'Measurement: An Essay in Philosophy of Science', *Theoria*, 38 (1972).

Asp; (ii) m is a measure for **D**; and (iii) m is a homomorphism from **Asp** onto a substructure of **S**.

Now, we can often give an explication of a quantity by identifying it with an aspect of our domain of individuals **D** and setting about the procedure of measurement along the lines suggested. This is certainly so in the case of height, which was one of the more plausible cases of reduction to predicates holding at a single time, so that even though (16) might be represented as (17), this by no means amounts to an elimination of the predicate 'is taller than' since it appears elsewhere in the analysis of the measure assigning height.

Consider now an example used earlier:

(10) John will repair the vase that Mrs. Smith broke.

which can be expressed in the form $\Gamma^{\alpha}\Pi^{\beta\alpha\beta}Aab$ (see (11)). The tensed predicate $\Gamma^{\alpha}\Pi^{\beta\alpha\beta}\ A_,_$ occurring here has the *prima facie* unusual characteristic of ascribing a relational property, which we might express as '_ repairs the vase that _ breaks', to two individuals relative to two times. Reichenbach would call such a tensed predicate a comparison between reference points. I want to extend this terminology so that a predicate like ${}^{\alpha\beta}A_,_$ above is regarded as a comparison, and in general predicates with two or more moment variables will be called comparisons. In the case of (10), the comparison there can be seen to comprise predicates ascribing properties holding at a single time (see (11)). But since, as has just been argued, the relational character of some predicates like 'is taller than' and 'is healthier than' cannot be eliminated, we ought to consider comparisons at the atomic level. Thus, to express (16) we need an atomic expression of the kind ${}^{\alpha\beta}Pxy$, for 'x is taller than y', and then (16) can be expressed in the form

(18) $\Delta^{\alpha}\Pi^{\beta\alpha\beta}Paa$.

This is not, of course, to deny the possibility of further explicating (16) along the lines of (17), but only to deny that such an explication obviates the need for the comparison appearing in what Kanger calls an aspect.

Perhaps it might be objected to this account that the relations appearing in the aspect contain only n-tuples of individuals compared at the same time and it is only in this way that the measure depends upon those relations, for (so the objections might run) it is empirically possible only to compare coexisting individuals. Any extension of these relations to cover comparisons across time must depend upon the measure in some such way as the following: Suppose we set up a system of measurement for height along the lines Kanger suggests, where relations in the aspect comprise only n-tuples of individuals compared at a single time. To extend, say, the 'taller than' relation, to cover what is expressed in (16), we suppose that I was, at the appropriate point in the past, compared with a rod and assigned a height at that time corresponding to the length in inches from one end of the rod to the point level with the top of my head. I am now compared with the same rod and assigned a height in the same way, and it is only on the ground that the number

first assigned to me is less than that later assigned to me that I can assert that I am taller than I was. Thus, (so the objection concludes) atomic comparisons with distinct moment variables are not required in the definition of the measurement procedure, but can themselves be defined in terms of the measure.

It seems to me, however, that this objection cannot be sustained since a comparison has been tacitly invoked in this procedure. How do we know that any height assigned by comparison with an interval on the rod, specified by one end of the rod and a notch, say, at any one time, is the same as that assigned at another time by comparison with the same notch? If the assignments are the same, that must presuppose that the length of the interval between the end of the rod and the notch at one time is the same as the interval between these points at the other time, and this clearly involves a comparison. Specifically, a predication of the form $^{\alpha\alpha\beta\beta}Pxyxy$ must be true, which reads 'the interval between x and y is the same as that between x and y', where the moment variables are appropriately bound or uniformly replaced by dates. It may well be that this particular comparison is itself defined, but it would be defined in terms of other comparisons. Perhaps it would clarify matters if this were spelt out in a little more detail.

Suppose, for the sake of illustration, that the aspect height is what Kanger calls a quasi-intensity – that is, an aspect for which there is a measure with respect to an interval scale. An interval scale is a structure $\mathbf{S} = (\mathbf{R},=,>,\mathbf{I}, \ldots)$ where $\mathbf{I}$ is a four-place relation such that, for each $i,j,k,l \in \mathbf{R}$, $\mathbf{I}ijkl$ if and only if $|i-j| \leqslant |k-l|$. then the aspect would have the form **Asp** $(\mathbf{U},\mathbf{V}(P_1), \mathbf{V}(P_2), \mathbf{V}(P_3), \ldots)$ with $^{\alpha\beta}P_1xy$ interpreted as 'x is the same height as y', $^{\alpha\beta}P_2xy$ as 'x is taller than y', and $^{\alpha\beta\gamma\delta}P_3xyzw$ as 'the difference in height between x and y is not greater than that between z and w', and the measure $^{\alpha}hx$ would satisfy the following conditions:

(i) $\mathbf{V}(h)$ is a binary function defined for each time and each individual in $\mathbf{U} \subseteq \mathbf{D}$;

(ii) the image set of h (and $\mathbf{V}(h)$) is a subset of $\mathbf{R}$; and

(iii) $\mathbf{M} \models {}^{\alpha\beta}P_1xy$ [s] iff $\mathbf{V}(h)(\mathbf{s}^*(\alpha),\mathbf{s}^*(x)) = \mathbf{V}(h)(\mathbf{s}^*(\beta),\mathbf{s}^*(y))$
$\mathbf{M} \models {}^{\alpha\beta}P_2xy$ [s] iff $\mathbf{V}(h)(\mathbf{s}^*(\alpha),\mathbf{s}^*(x)) > \mathbf{V}(h)(\mathbf{s}^*(\beta),\mathbf{s}^*(y))$
$\mathbf{M} \models {}^{\alpha\beta\gamma\delta}P_3xyzw$ [s] iff $\mathbf{I}(\mathbf{V}(h)(\mathbf{s}^*(\alpha),\mathbf{s}^*(x)),\mathbf{V}(h)(\mathbf{s}^*(\beta),\mathbf{s}^*(y)),$
$\mathbf{V}(h)(\mathbf{s}^*(\gamma),\mathbf{s}^*(z)),\mathbf{V}(h)(\mathbf{s}^*(\delta),\mathbf{s}^*(w)))$,

where $\mathbf{V}(h)$ is the function assigned to h by the valuation $\mathbf{V} \in \mathbf{M}$. Strictly speaking, this last clause does not make a great deal of sense since the notion of satisfaction has not yet been extended to cover the language enlarged by the inclusion of comparisons. However, the valuation function $\mathbf{V}$ will be extended to cover the new cases shortly. Meanwhile, these clauses can be taken to express some desiderata for an adequate extension of $\mathbf{L}^*$ and $\mathbf{V}$.

The expression $^{\alpha\alpha\beta\beta}Pxyxy$ for 'the interval between x and y is the same as that between x and y' mentioned above can now be defined as $^{\alpha\alpha\beta\beta}P_3xyxy$ & $^{\beta\beta\alpha\alpha}P_3xyxy$. That such a comparison should hold in cases like that of our stan-

dard rod mentioned above must be included as part of what Kanger calls the specification of the aspect **Asp** of **D**. If it is held that this is not ascertainable by empirical means, then this part of the specification will not be operational, as Kanger would say, but rather must be laid down by stipulation. In any case, comparisons play an essential role in comparing measurements of height made at different times.

4. The Language L

In this section we consider precisely how to extend L* by the introduction of comparisons, and concomitantly, how to extend the semantic machinery developed for L* to cope with the extended language L.

We have seen in the course of the previous section examples of comparisons with two and with four moment variables. In general we can introduce atomic formulae of the form $^{u_{i_1} \ldots u_{i_n}}P_j^n\ u_{k_1} \ldots u_{k_n}$, where $u_{i_1}, \ldots, u_{i_n}$ are moment terms and $u_{k_1}, \ldots, u_{k_n}$ are indivudal terms. Atomic predications of the original kind can be regarded as special cases where the moment terms are identical, so that we have

D6 $u P_j^n u_{k_1} \ldots u_{k_n} =_{\text{df}} {}^{u_{i_1} \ldots u_{i_n}}P_j^n u_{k_1} \ldots u_{k_n}$, where $u = u_{i_1} = \ldots = u_{i_n}$

As for the semantics of these new predicates, two possibilities seem natural for the extension $\mathbf{R}_j^n$ of the new predicate letters, namely

(i) $\mathbf{R}_j^n$ is a subset of $(\mathbf{T} \times \mathbf{D})^n$, so that $\mathbf{R}_j^n$ is a set of n-tuples of pairs (t,d) with $t \epsilon \mathbf{T}$ and $d \epsilon \mathbf{D}$; and

(ii) $\mathbf{R}_j^n$ is a subset of $\mathbf{T}^n \times \mathbf{D}^n$, so that $\mathbf{R}_j^n$ is a set of 2n-tuples $(t_{i_1}, \ldots, t_{i_n}, d_{i_1}, \ldots, d_{i_n})$ with $t_{i_j} \epsilon \mathbf{T}$ and $d_{i_j} \epsilon \mathbf{D}$ $(j = 1,2, \ldots, n)$.

However, since continuity with molecular expressions was emphasised in developing the notion of an atomic comparison, it is appropriate that considerations of the same sort should bear on the choice made here in the semantics, and these considerations favour (ii). For if we consider how to extend $\mathbf{R}_j^n$ so that we can assign a relation to any wff $^{\alpha_{i_1} \ldots \alpha_{i_m}} A x_{j_i} \ldots x_{j_n}$ of L* with m distinct free moment variables and n distinct free individual variables, and so that in the special case that A is $^{\alpha_i}P_j^n x_{j_1} \ldots x_{j_n}$ the extension conforms to $\mathbf{R}_j^n$ as previously defined, a natural definition of $\mathbf{R}(A)$, relative to our interpretation **M**, is

$$\mathbf{R}^{m,n}(A) =_{\text{df}} \{(s_{i_1}, \ldots, s_{i_m}, s_{j_1}, \ldots, s_{j_n}) : \mathbf{M} \models A\,[\mathbf{s}]\}$$

In general, the number of moment variables will not equal the number of individual variables, so that a definition of $\mathbf{R}^{m,n}(A)$ along the lines of (i) with each individual paired with a time, and vice versa, is not possible.

On this account, (ii) comes out as a special case of $\mathbf{R}^{n,n}(A)$ where A is an atomic formula and $m=n$. One might wonder whether this last restriction on atomic formulae is really necessary, but I have not been able to find convincing examples of atomic comparisons with a greater or lesser number of moment terms than individual terms. Two examples do spring to mind, namely the 'earlier than' relation E and the identity relation; but in the absence of further good examples I do not think we would be justified in generalising atomic predicates so that these become special cases when there are no individual terms or no moment terms.

Finally, then, we define L as like L* except that part (i.1) of definition D2 is replaced by a clause to the effect that, if P_j^n is in **Pred**, $u_{i_1}, \ldots, u_{i_n}$ are moment terms, and $u_{j_1}, \ldots, u_{j_n}$ are individual terms, then $u_{i_1} \ldots u_{i_n} P_j^n u_{j_n} \ldots u_{j_n}$ is an atomic formula. As for the semantics, the valuation function is changed so that part (iv,5) of definition D3 is replaced by a clause to the effect that if P_j^n is in **Pred**, $\mathbf{V}(P_j^n)$ is a subset $\mathbf{R}_j^n$ of $\mathbf{T}^n \mathbf{x} \mathbf{D}^n$.

5. Tenses

The formulation of tensed statements expressed in English has been discussed at some length. Now that our formal system is fully developed, I shall try to make the notion of a tense at issue throughout this discussion more succinct by describing precisely which expressions of L are to count as tenses and outlining a procedure for ascribing tenses to formulae of L. It will be possible, on the basis of this procedure, to define the categories of tenses indicated in the classification outlined at the beginning of this chapter.

The procedure according to which tenses are ascribed to a formula depends upon the possibility of transforming it into a logically equivalent formula in prenex normal form, so that logically equivalent formulae will be ascribed the same number of tenses in this way. A formula is said to be in *prenex normal form* when all the quantifiers occur at the front so that it has two components: a *prefix*, which is just a string of quantifiers, and a *matrix* which is the remaining subformula containing no quantifiers. This definition is somewhat different from the usual one given for first-order languages since indexical quantifiers are to count as quantifiers here. Moreover, we shall also require in our procedure for ascribing tenses to formulae that the formula in question be abbreviated as far as possible according to DfF, DfP, DfG and DfH, and the expressions introduced in this way are to count as quantifiers falling within the prefix. This seems to me a reasonable extension of the usual definition in view of lemma 6.

Within the prefix I shall distinguish sequences of quantifiers to be called *chains*. These are sequences of the form $\mathbf{W}^{\beta_1}\beta_1 Q_1{}^{\beta_2} \ldots {}^{\beta_n} Q_n^{\gamma}$, where each Q_i ($i = 1,2, \ldots, n$) is F,P,G or H. The quantifiers of the chain need not be adjacent as they occur in the prefix; indeed, as will become apparent in the next chapter, this would sometimes be impossible. But it is required that, when n is greater than 1, the

anterior variable of Q_i (i.e., β if Q_i occurs as ${}^{\beta}Q_i^{\delta}$), where i is greater than 1, be bound by the *posterior variable* of Q_{i-1} (i.e., β if Q_{i-1} occurs as ${}^{\alpha}Q_{i-1}^{\beta}$). The special cases where $n = 0$ and we have just an indexical quantifier are counted as chains. A tense will be said to comprise such a chain, though not without further conditions which will be introduced presently. First, we must show that there is an effective procedure for transforming any wff of L into a logically equivalent formula in prenex normal form, as this notion is defined here.

We already have some of the preliminary results used in the proof of the prenex normal form theorem, namely theorem 5 and corollary 7.1. Furthermore, we have already noted that L* is extensional in the sense that any formula is transformed into a logically equivalent formula by replacing any subformula with one of its logical equivalents. These results, and for that matter all of the others adduced so far, also hold for L. The additional lemmas that will be required are as follows:

Lemma 8

If γ is not free in A or B, and β is free for γ in A, then the following equivalences are valid:

(i) $({}^{\alpha}G^{\beta}A \vee B) \equiv {}^{\alpha}G^{\gamma}((\gamma/\beta)A \vee B)$

(ii) $({}^{\alpha}F^{\beta}A \vee B) \equiv {}^{\alpha}F^{\gamma}((\gamma/\beta)A \vee B)$

(iii) $({}^{\alpha}H^{\beta}A \vee B) \equiv {}^{\alpha}H^{\gamma}((\gamma/\beta)A \vee B)$

(iv) $({}^{\alpha}P^{\beta}A \vee B) \equiv {}^{\alpha}P^{\gamma}((\gamma/\beta)A \vee B)$.

Proof

(i) $\mathbf{M} \models ({}^{\alpha}G^{\beta}A \vee B)$ [s]

iff $\mathbf{M} \models {}^{\alpha}G^{\beta}A$ [s] or $\mathbf{M} \models B$ [s]

iff $\mathbf{M} \models {}^{\alpha}G^{\gamma}(\gamma/\beta)A$ [s] or $\mathbf{M} \models B$ [s], by corollary 2.1

iff $\mathbf{M} \models (\gamma/\beta)A$ $[(t/\gamma)\mathrm{s}]$ or $\mathbf{M} \models B$ $[(t/\gamma)\mathrm{s}]$, for every $t \epsilon \mathbf{T}$ s.t. $\mathrm{s}^*(\alpha) < t$, by lemma 6 and lemma 1

iff $\mathbf{M} \models {}^{\alpha}G^{\gamma}((\gamma/\beta)A \vee B)$ [s]

(ii), (iii) and (iv) follow along similar lines.

Lemma 9

(i) $\models \sim \Lambda v A \equiv \mathrm{V} v \sim A$

(ii) $\models \sim \mathrm{V} v A \equiv \Lambda v \sim A$

and if v_i is not free in A or B and v_j is free for v_i in A, then

(iii) $\models (\Lambda v_j A \vee B) \equiv \Lambda v_i((v_i/v_j)A \vee B)$

(iv) $\models (\mathrm{V} v_j A \vee B) \equiv \mathrm{V} v_i((v_i/v_j)A \vee B)$.

(i) and (ii) are familiar; (iii) and (iv) follow by proofs similar to that exemplified for lemma 8.

Theorem 10

There is an effective procedure for transforming any wff A into a wff B in prenex normal form logically equivalent to A.

Proof: By induction on the length of A.
Case 1 (Basis). A is an atomic formula. Then B is A.
Case 2: A is $\sim C$. By induction hypothesis, we can construct a wff D, logically equivalent to C, in prenex normal form. Hence, $\sim C$ is logically equivalent to $\sim D$, and then by corollary 7.1, parts (i) and (ii) of lemma 9, and theorem 5, we can construct a wff B in prenex normal form logically equivalent to $\sim D$, and hence, to A.
Case 3: A is $C \mathbf{v} D$. By induction hypothesis, we can construct wffs C_1 and D_1 logically equivalent, respectively, to C and D, in prenex normal form. Thus, $C \mathbf{v} D$ is logically equivalent to $C_1 \mathbf{v} D_1$ and by applying theorem 5, lemma 8 and parts (iii) and (iv) of lemma 9, the quantifiers in the prefixes of C_1 and D_1 can be moved to the front giving us a wff B, logically equivalent to A, in prenex normal form.
Case 4: A is $\Lambda \nu C$. By induction hypothesis, there is a wff D, logically equivalent to C, in prenex normal form, so that A is logically equivalent to $\Lambda \nu D$, and this formula is now in prenex normal form.
Case 5: A is $\mathbf{W}^{\alpha} C$, and the proof proceeds as in case 4.

Note that wherever indexical quantifiers occur in subformulae of A, they are moved into the prefix by applications of theorem 5 rather than lemmas 3 and 4. This has the effect of reducing the number of occurrences of indexical quantifiers so that in the prefix B there is at most one occurrence of each of them. By further applications of theorem 5, any indexical quantifiers could be moved to the front of the prefix, but this will not be necessary for our present purposes.

Now we are in a position to consider under what conditions a chain might reasonably be regarded as a tense, and then to set out the procedure for characterising the tenses occurring in a formula of L. The point of the procedure for characterising the tenses in a formula is to provide a principle for counting the tenses in accordance with our intuitions on this matter, and so that logically equivalent formulae are assigned the same number of tenses.

We might begin with the suggestion that the number of tenses ascribed to a formula would be unnecessarily large if by a tense we understood simply a chain as outlined above. For if any chain $\mathbf{W}^{\beta_1 \beta_1} Q_1{}^{\beta_2} \ldots {}^{\beta_n} Q_n^{\gamma}$ were to count as a tense, so would the n expressions $\mathbf{W}^{\beta_1}, \mathbf{W}^{\beta_1 \beta_1} Q_1{}^{\beta_2}, \ldots,$ and $\mathbf{W}^{\beta_1 \beta_1} Q_1{}^{\beta_2} \ldots {}^{\beta_{n-1}} Q_n^{\beta_{n-1}}$. To rectify this, we might say that a tense is a chain which is *complete* (with respect to the formula in which it occurs) in the sense that, if β is the posterior variable of the last quantifier in the chain, there is no quantifier falling within the scope of the chain with an anterior variable β. However, this would be too restrictive and conflict with our intuitions. Consider, for example, an illustration from the section on sequences of tenses, namely 'He had telephoned when he came'. We expressed this statement in the form $\Pi^{\alpha}({}^{\alpha}P^{\beta\beta} Ba \,\&\, {}^{\alpha}Aa)$, which on the present suggestion would be said to have two tenses (a past posterior and a simple past) – all well and good. But the formula becomes $\Pi^{\alpha\alpha} P^{\beta}({}^{\beta}Ba \,\&\, {}^{\alpha}Aa)$ in prenex normal form, and this formula – logically equivalent to the first – would be said to have just one tense on this view of tenses, and this hardly seems natural. I think it is clear from

this example that the notion of tense we are after does not depend entirely on the quantifiers present in a formula, but on the structure of the atomic constituents as well. Let us say that a chain binds a variable α if α is the posterior variable of the last member of the chain. Then, when the formula in question is in prenex normal form, the matrix can be thought of as a comparison, and a tense as any chain which binds a moment variable within the matrix. These moment variables in the matrix need not be distinct, so that a formula of the kind $\Pi^{\alpha}({}^{\alpha}A \,\&\, {}^{\alpha}B)$, for example, where ${}^{\alpha}A$ and ${}^{\alpha}B$ are atomic formulae, will be said to have two tenses. However, if ${}^{\alpha}A$ or ${}^{\alpha}B$ here are atomic comparisons, but containing just one distinct moment variable, I do not think it would be apposite to say that the formula contained more than two tenses, so when assigning tenses to a formula we shall require that the matrix be abbreviated as far as possible according to D6.

We already have the definition of a chain; the definition of a tense and the procedure for distinguishing tenses within a formula can now be summarised as follows:

D7 *Tenses*

The tenses occurring in a wff *A* of L are characterised by the following procedure: Construct the wff *B* obtained from *A* by inspecting its successive subformulae and applying Df*P*, Df*F*, Df*H*, Df*G* and D6 wherever possible. Then construct the wff *C* in prenex normal form logically equivalent to *A* according to theorem 10. For every *occurrence* of a moment variable in the matrix of *C*, a chain binding that moment variable in that occurrence is counted as a tense of *A* (even if that chain has already been counted with respect to another occurrence of the same variable).

At the beginning of Chapter III we said that, on the view adopted here, tenses are an integral part of the predicate occurring in a statement. This can be brought out a little more by defining what kind of expressions are to count as tensed predicates in our language.

D8 *Tensed predicates*

Let $\mathbf{U}^n$ be the expression ${}^{u_1}\ldots{}^{u_n}P_j^n$, where P_j^n is a predicate letter and $u_1, \ldots, u_n$ are moment terms. Suppose V is a string of moment quantifiers. Then the expression $\mathbf{VU}^n$ is said to be a *tensed (atomic) predicate* if the expression $\mathbf{VU}^n v_1 \ldots v_n$, where $v_1, \ldots, v_n$ are individual terms, is a wff of L containing no free moment variables but at least one tense.

Similarly, we can define a tenseless predicate as follows:

D9 *Tenseless predicates*

Let $\mathbf{U}^n$ be the expression ${}^{u_1}\ldots{}^{u_n}P_j^n$, where $u_1, \ldots, u_n$ are moment terms and P_j^n is a predicate letter or the expression E or $=$. Suppose V is a string of zero or more moment quantifiers. We say that the expression $\mathbf{VU}^n$ is a *tenseless (atomic) predicate* if the expression $\mathbf{VU}^n v_1 \ldots v_n$, where $v_1, \ldots, v_n$ are moment or

individual terms, is a wff of L containing no free moment variables and no indexical quantifiers.

The notions of a tensed and a tenseless statement which figured in earlier chapters can also be defined.

D10 *Tensed statements*

A wff A is said to be a *tensed statement* if it contains no free moment variables and at least one tense.

D11 *Tenseless statements*

A wff A is said to be a *tenseless statement* if it contains no free moment variables and no tenses.

Formulae containing free occurrences of moment variables are not classified as either tensed or tenseless statements, and are not used as formulations of statements made in English. Formulae like Em_1m_2, $^{m}P_j^1a$ and $\Lambda\alpha\,{}^{\alpha}P_j^1a$ are classified as tenseless statements according to these definitions, and intuitively this seems to be correct; but formulae like $\Pi^{\alpha}({}^{\alpha}P_j^1a \,\&\, \alpha{=}m)$ containing both a tense and a date, might not seem so clear-cut and it might be thought an arbitrary decision whether this statement is called tensed, tenseless or both tensed and tenseless. However, if we consider that the formula in question could be used to represent the statement 'Bertrand Russell died on 2nd February 1970', it strikes me as more in accordance with our intuitions to classify this as a tensed statement, and this policy is reflected in the definitions.

The rudimentary schemes for the classification of tenses outlined in earlier sections of this chapter can be set out as follows:

D12 *Simple tenses*

Tenses of the kind $\mathbf{W}^{\beta}$ are said to be *simple tenses*.

D13 *Complex tenses*

Tenses of the kind $\mathbf{W}^{\beta_1\beta_1}Q_1^{\beta_n}\cdots{}^{\beta_n}Q_n^{\gamma}$ with n greater than O are called *complex tenses*. The special cases where $n=1$ are called *anterior tenses* when Q_1 is P, and *posterior tenses* when Q_1 is F.

D14 *Past, present and future tenses*

A tense is a *past (present, future) tense* if the initial indexical quantifier is $\Pi(\Delta,\Gamma)$.

Chapter V

Quantification over Individuals in Tensed Contexts

1. A Comparison of Priorean Tense Logic with L

The pioneering work of Prior on the construction of tense logics has featured prominently in the first two chapters leading up to the presentation of our formal language L with its associated semantics. We have seen that Prior's analysis of tenses was developed in the spirit of McTaggart's views on tenses. But it has been our expressed intention to oppose those views and to build up a systematic account of temporal reference on the basis of a different interpretation of tenses. Now that we have the formal system L with the associated semantics, together with some account of how the formal system is intended to be used to express statements made in English, the time has come to see whether these differences in interpretation are sustained in the formal systems and issue in a difference in expressive power of the two systems. Comparing the expressive powers of the two systems will take the form here of investigating whether there are statements that can be made in English which can be represented satisfactorily in one system but not the other. Thus, the criterion of acceptability of the interpretation of tenses proposed here is to be that it leads to an adequate account of the usage of tenses in English, or at least, a more adequate account than the Priorean interpretation provides.

It is possible to build up constructions in our system which are similar to the sequences of iterated tense operators *P,H,F* and G of Prior's systems, namely constructions like ${}^{\alpha}F^{\beta\beta}G^{\gamma}$, ${}^{\alpha}F^{\beta\beta}F^{\gamma\gamma}F^{\delta}$, etc., preceded by an indexical quantifier binding the intitial moment variable α. Thus, a statement like 'John will have been going to come', which Prior would no doubt put in the form *FPFA*, might be expressed in our system by $\Delta^{\alpha\alpha}F^{\beta\beta}P^{\gamma\gamma}F^{\delta\delta}A$. So there is a substantial similarity between the two kinds of tense logic, even though it has been our intention to develop a different interpretation of tenses. Our immediate task, then, is to try to express this similarity more concisely to facilitate the comparison of the two kinds of system.

We shall suppose that the language of conventional tense logics is constructed from the primitives Λ,∼,**v**,*P,F,* (and), a set of predicate letters, a set of individual variables and a set of individual constants in the usual way. It is possible to set up a procedure for translating any arbitrary formula of this Priorean language, as we shall sometimes refer to it, into a wff of L in a natural and illuminating way as

follows. All individual variables and constants remain unchanged. Every wff A of the Priorean language is translated into a wff of L of the form $\Delta^{\alpha\alpha}A$, where ${}^{\alpha}A$ is defined by recursion thus:

I $\quad {}^{\alpha}(P_j^n u_1 \ldots u_n)$ is ${}^{\alpha}P_j^n u_1 \ldots u_n$

II (i) ${}^{\alpha}(\sim A)$ is $\sim {}^{\alpha}A$

(ii) ${}^{\alpha}(A \vee B)$ is ${}^{\alpha}A \vee {}^{\alpha}B$

(iii) ${}^{\alpha}(\Lambda x A)$ is $\Lambda x {}^{\alpha}A$

(iv) ${}^{\alpha}(PA)$ is ${}^{\alpha}P^{\beta\beta}A$, where β is a moment variable not hitherto appearing in the translation.

(v) ${}^{\alpha}(FA)$ is ${}^{\alpha}F^{\beta\beta}A$, where β is a moment variable not hitherto appearing in the translation.

Using $\rightarrow$ as an abbreviation for 'is translated under this procedure as', the following are examples of the application of this procedure:

(1) $FPFA \rightarrow \Delta^{\alpha\alpha}F^{\beta\beta}P^{\gamma\gamma}F^{\delta\delta}A$

(2) $F(PA \vee FB) \rightarrow \Delta^{\alpha\alpha}F^{\beta}({}^{\beta}P^{\gamma\gamma}A \vee {}^{\beta}F^{\delta\delta}B)$

(3) $FVx(PA \,\&\, FB) \rightarrow \Delta^{\alpha\alpha}F^{\beta}Vx({}^{\beta}P^{\gamma\gamma}A \,\&\, {}^{\beta}F^{\delta\delta}B)$

The Priorean formula in the last example has been expressed in primitive notation, translated, and the resulting translation has been expressed with the same abbreviations used in the original Priorean formula. We shall continue to use $\rightarrow$ in this extended sense so that abbreviations common to the two languages remain unaltered.

Clearly, every Priorean formula can be mapped onto a distinct wff of L in this way, so that the mapping is not many one. As for translation in the converse direction, this may not always be possible. Suffice it to say for the present that since neither Π nor Γ appear in the translation procedure, and nor do any atomic comparisons, it follows immediately that the translation procedure maps the Priorean language onto a proper subset of L. I shall expand on this fact later; for the moment I want to consider only those formulae of L which do correspond to the Priorean language in this way and bring out some differences of interpretation here.

All formulae in this subset of L corresponding to the Priorean formulae are present tensed according to our account, which so far accords with Prior's reading all his formulae as present tensed. However, unlike the Priorean formulae, the members of this subset of L are said to have no past or future tense components over and above the present tense. Rather, inclusion of the expression ${}^{\alpha}P^{\beta}$ and ${}^{\alpha}F^{\beta}$ serves to distinguish different kinds of present tense, some of which we have given names; for example, the simple present and the present anterior. The existential quantifiers contained in ${}^{\alpha}F^{\beta}$ and ${}^{\alpha}P^{\beta}$ were introduced, in fact, in connection with

Diver's notion of indefinite tenses, and we could say that Prior has in his language, apart from formulae containing no tense operators, only expressions of indefinite tenses. But perhaps the most important consequence of our view of the present tenses arises from the principle that the point of view of the speaker dominates all subordinate contexts, and the difference this gives rise to between Prior's interpretation of formulae containing iterated tense operators and our interpretation of the corresponding translations. A formula like *FPFA*, for example, we are told to read as '*A* is in the future past future'; but such a reading is not at all plausible for the translation $\Delta^{\alpha\alpha}{}_{\alpha}F^{\beta\beta}{}_{\beta}P^{\gamma\gamma}{}_{\gamma}F^{\delta\delta}A$. For although the first occurrence of ${}^{\alpha}F^{\beta}$ takes us to some time in the future, there is no indication of whether ${}^{\beta}P^{\gamma}$ then takes us into the past, rather than back to the present or to a time less far into the future than the first time. Similarly, it is not clear whether the expression ${}^{\gamma}F^{\delta}$ then takes us into the future, for it is possible that this might leave us in the past or the present if the preceding expression ${}^{\beta}P^{\gamma}$ actually took us into the past. This is because the division of time into past, present and future remains unchanged in all subordinate contexts since what is taken for the present moment remains the same, and so the phrase 'future past future' is just not appropriate to the reading of our formula.

These comments bring out a difference in interpretation of statements made in English for which there is always a candidate from both the Priorean language and L as the expression of its logical form, but this is already familiar from previous chapters. Now I shall try to further substantiate our interpretation of tenses by showing that it leads to a more adequate account of tenses in English in virtue of the greater expressive power of L compared with the Priorean language.

We have seen already that the Priorean language corresponds under translation to only a proper subset of L since certain primitives of L do not appear in the translation procedure. But even within that subset of L where just those symbols used in the translation procedure appear, there remain formulae which are not the translations of any formula of the Priorean language. To see this, it is important to notice first that any Priorean formula is not longer than its translation. (In fact, a Priorean formula is shorter than its translation since, although the number of individual terms and predicate letters remains unchanged, the number of operators is increased by at least one in view of the introduction of Δ, and by more than one if the expressions ${}^{\alpha}P^{\beta}$ or ${}^{\alpha}F^{\beta}$ are introduced, since they give rise to more operators when expanded in primitive terminology.) Consequently, for any wff of L it is possible to enumerate a finite number of Priorean formulae comprising all those wffs which can be formed from connectives and quantifiers of the Priorean language whose number does not exceed the number of connectives and quantifiers in the wff of L. Inspection of the finite list will eventually yield an answer to the question whether any given formula of L is the translation of a Priorean formula, and in practice this finite list can be reduced to a small set of the more plausible candidates very quickly. Now consider a formula like

(4) $\Delta^{\alpha\alpha}P^{\beta}\forall x({}^{\beta}A \;\&\; {}^{\alpha}F^{\gamma\gamma}B)$

of L, which expresses the form of, say, 'Some people who have infiltrated the party will try to destroy it'. Of the appropriate finite list of Priorean formulae there are just two candidates worth considering for which (4) might be a translation:

(5) $PVx(A \,\&\, FB) \rightarrow \Delta^{\alpha\alpha}P^{\beta}Vx(^{\beta}A \,\&\, {}^{\beta}F^{\gamma\gamma}B)$

(6) $Vx(PA \,\&\, FB) \rightarrow \Delta^{\alpha}Vx(^{\alpha}P^{\beta\beta}A \,\&\, {}^{\alpha}F^{\gamma\gamma}B)$.

However, since neither of these is translated as (4), (4) is not the translation of any Priorean formula. Consequently, the translation procedure maps the Priorean language onto a proper subset of that fragment of L containing only those primitives appearing in the translation procedure.

This, however, is not sufficient to establish our thesis of the greater expressive power of L. For the formula of L appearing in (6) is in fact logically equivalent to (4) (provided there are no occurrences of β in B, which there need not be if (4) is to represent the form of 'Some people who have infiltrated the party will try to destroy it'). So although (4) is not itself the translation of any Priorean formula, we have not yet shown that it expresses anything which cannot be expressed by a Priorean formula. But we are now in a better position to state clearly what criteria should be met in order to show that this fragment of L has a greater expressive power than the Priorean language. We have to find at least one member of this fragment of L which (a) is not the translation of any Priorean formula, (b) is not equivalent to any formula of L which is the translation of a Priorean formula and (c) can be shown to have some application in expressing statements that can be made in English.

Intuitively, any such gain in expressive power will reflect what Prior took to be the unusual property of 'now', that it takes the whole statement in which it occurs as its scope (he speaks of "this deplorable holism" in this connection[1]), and any formula we can find satisfying the requirements (a), (b) and (c) will convincingly establish the non-redundancy of an explicit indication of the present tense. Consider, then, the statement

(7) Some day, all those who took part in the plot will be brought to justice,

which I express in the form

(8) $\Delta^{\alpha\alpha}F^{\beta}\Lambda x(^{\alpha}P^{\gamma\gamma}Ax \supset {}^{\beta}Bx)$,

where Ax stands for 'x takes part in the plot' and Bx for 'x is brought to justice'. There are just two serious candidates amongst the Priorean formulae which might express (7) correctly, but it turns out that neither of their translations is (8). Thus

(9) $F\Lambda x(PAx \supset Bx) \rightarrow \Delta^{\alpha\alpha}F^{\beta}\Lambda x(^{\beta}P^{\gamma\gamma}Ax \supset {}^{\beta}Bx)$

(10) $\Lambda x(PAx \supset FBx) \rightarrow \Delta^{\alpha}\Lambda x(^{\alpha}P^{\beta\beta}Ax \supset {}^{\alpha}F^{\gamma\gamma}Bx)$.

[1] Prior, *Papers on Time and Tense*, p. 23.

and so requirements (a) and (c) are satisfied. (Perhaps we should mention some more possible formulations in the Priorean language to make our case more convincing; for example, $\Lambda xF(PAx \supset Bx)$ and $\Lambda xP(Ax \supset FBx)$. But (8) is not the translation of either of these, and should anyone feel that one of these expresses (7) adequately, it will become apparent why this cannot be maintained presently.) Now, neither of the formulae of L appearing in (9) and (10) is logically equivalent to (8), and so condition (b) is also satisfied. This can be seen intuitively. For in the case of (9), the expression ${}^{\beta}P^{\gamma}$ does not necessarily take us to a moment in the past since the occurrence of β here is bound by $\Delta^{\alpha\alpha}F^{\beta}$, whereas the corresponding expression ${}^{\alpha}P^{\gamma}$ as it occurs in (8) most certainly does take us to a moment in the past. And in the case of the formula of L appearing in (10), this is compatible with each person who took part in the plot being brought to justice at different times in the future, whereas this is not compatible with (8), and so (8) is not logically equivalent to that formula. (In the one case, the Priorean formula is inappropriate because a tense operator falls within the scope of another, and in the other case, because the universal quantifier occurs outside the scope of a tense operator. The Priorean formulae mentioned above, namely $\Lambda xF(PAx \supset Bx)$ and $\Lambda xP(Ax \supset FBx)$, are inappropriate for both these reasons.)

Again, consider the statement

(11) There will always be somewhere in space some remnants of the particles formed in the primordial explosion,

which we express in the form

(12) $\Delta^{\alpha\alpha}G^{\beta}\forall x({}^{\alpha}P^{\gamma\gamma}Ax \,\&\, {}^{\beta}Sx)$,

where Ax stands for 'x is a remnant of some particle formed in the primordial explosion' and ${}^{\beta}Sx$ stands for 'x is somewhere in space at β'. (I shall have a little more to say about this predicate S later.) Once again, it will be sufficient to consider just two Priorean formulae in order to bring out the limitations in the resources of the Priorean language which preclude the possibility of expressing (11) adequately there; thus

(13) $G\forall x(PAx \,\&\, Sx) \rightarrow \Delta^{\alpha\alpha}G^{\beta}\forall x({}^{\beta}P^{\gamma\gamma}Ax \,\&\, {}^{\beta}Sx)$

(14) $\forall x(PAx \,\&\, GSx) \rightarrow \Delta^{\alpha}\forall x({}^{\alpha}P^{\gamma\gamma}Ax \,\&\, {}^{\alpha}G^{\beta\beta}Sx)$.

Neither of the formulae of L appearing here is identical with (12), and nor is either equivalent to (12). Informally, in the case of (13) this is because the P operator of the Priorean formula occurs within the scope of the G operator, and so for each moment in the future, takes us to a time earlier than that. The corresponding operator ${}^{\alpha}P^{\gamma}$ appearing in (12), on the other hand, takes us to a time earlier than the present. In the case of (14), the G operator of the Priorean formula occurs within the scope of the existential quantifier, so that for this formula to be satisfied there must be some remnant of the primordial matter which will always retain its

identity as that particular remnant, whereas (12) allows that at different times in the future there might be different remnants of this matter. Consequently, neither of the formulae of L occurring in (13) and (14) is logically equivalent to (12) and we have another example satisfying all three conditions mentioned above.

These two examples, (7) and (11), illustrate very well why an explicit present tense operator is necessary for an adequate formalisation of tenses in English. For on the one hand, what is taken to be the present moment must be the same in all subordinate contexts so that it is possible to refer to the past, present or future from some fixed point of reference. (This is why the Priorean formulae appearing in (9) and (13) are unsatisfactory: the past tense operator P occurs within the scope of some other tense operator and so does not in fact take us necessarily to a point in the past.) And on the other hand, reshuffling a Priorean formula in an attempt to rectify problems arising on the first count sometimes disturbs the order of certain operators with a corresponding adjustment of meaning. (This is why the Priorean formulae appearing in (10) and (14) are unsatisfactory.)

2. Kamp's and Vlach's Extensions of the Priorean Language

In order to deal with some of these limitations of Priorean tense logic, proposals have been forthcoming by way of extending Prior's basic system, notably due to Hans Kamp.[2] Kamp has introduced a one-place sentential operator N intended to reflect the behaviour of the idiomatic 'now', so that formulae of the kind NA appear and are read 'it is now the case that A'. This new operator is adequate to cope with the problems arising for the Priorean language to which we drew attention in the previous section, although I have some reservations about how closely it reflects idiomatic usage which will appear in the next section. Apart from reservations of this kind, however, Kamp's extension of the Priorean language is still not adequate to deal with all tensed contexts in English, for we can find statements which cannot be satisfactorily formulated with the resources of his language. A further extension of the Priorean language, encompassing Kamp's extension, has in fact been proposed by Frank Vlach[3] to deal with a wider class of statements. A brief exposition of Kamp's and Vlach's proposal is given in this section, paving the way for a comparison between L and this extension of the Priorean language in the following section.

Kamp provides a semantic and syntactic characterisation of his extended Priorean system. The syntactic development takes the form of constructing, for any set **Ax** of axioms of the Priorean language which are semantically complete

[2] Hans Kamp, 'Formal Properties of "Now"', *Theoria*, 35 (1971). Further references to this article are indicated in the body of the text.

[3] Frank Vlach, *"Now" and "Then": A Formal Study in the Logic of Tense Anaphora*, Draft Ph.D. thesis to be submitted at the University of California in Los Angeles.

relative to a given ordering of moments, an extension **Ax**′ of **Ax** for the extended system which is semantically complete. However, the syntactic development will not be pursued here, for our interest lies primarily in the semantic characterisation. Kamp begins his semantic analysis with the suggestion that the concept we ought to analyse is not simply 'the truth value of A at t', but rather 'the truth value of A at t when part of an utterance made at t''' (p. 238). By keeping track of the moment of utterance during the evaluation of the truth value of a tensed proposition in this way, Kamp is able to reflect what Prior expressed as the principle that 'now' takes the whole utterance in which it occurs as its scope. Thus, retaining the idea that tensed propositions are propositions whose truth values may vary with time, the truth value of a proposition is now evaluated at a pair of times (t,t'), the second member t' being interpreted as the moment of utterance. Then, a formula of the kind PA is said to be true at t when part of an expression uttered at t' iff A is true at t'', still considered part of an utterance made at t', where t'' is some time earlier than t. The evaluation of formulae of the kinds FA, $\sim A$, $A \vee B$, etc., follows a similar pattern. A formula of the kind NA is said to be true at t when part of an expression uttered at t' iff A is true at the time of utterance of the expression of which it is part, t' (i.e., NA is true at (t,t') iff A is true at (t',t')).

Using formal semantics based on these ideas Kamp is able to show that, whilst for propositional tense logic the 'now' operator N is always eliminable – for every formula of propositional tense logic containing this operator there is a logically equivalent formula in which it does not occur—this is not so in quantificational tense logic where the N operator is not always eliminable. And our examples from the previous section which could not be formulated in the Priorean language can be expressed with the resources of this extended system. Thus, (7) might be expressed in the form:

(15) $F\Lambda x(NPAx \supset Bx)$,

and (11) in the form

(16) $GVx(NPAx \supset Sx)$.

Now, Vlach has suggested that the Priorean language be even further extended in order that it should reflect better the usage of tenses in English. He begins with the suggestion that not every formula in Kamp's system has a past tense. In the Priorean system, each formula A can be put in the past tense simply by prefixing the operator P (and similarly, the future tensed formula corresponding to A is FA), whereas in the case of a formula like (15), prefixing a P operator does not give what would, as Vlach claims, intuitively be taken as its past tensed modification. To see what Vlach is getting at here, consider (15) prefixed by a P operator

(17) $PF\Lambda x(NPAx \supset Bx)$.

Keeping the same interpretations of Ax and Bx, (17) would be read something like

(18) One day, all those who took part in the plot were going to be brought to justice.

But this, considered as a realisation of (17), allows the possibility that those taking part in the plot are brought to justice at a time before they actually participated in the plot! We have to ensure, in order to get a more natural past tense modification of (15), that at some point in the past, all those who took part in the plot before this time are brought to justice after this time. Thus, a point of reference in the past seems to be involved here, and given what has been said in earlier chapters about the correlation between the use of a reference point in the past and certain verb forms in English, we might express the realisation in English of the past tensed formula we are looking for by

(19) One day, all those who had taken part in the plot were going to be brought to justice.

Vlach formulates the notion of past tense appropriate here by defining a formula A as the *past tense* of a formula B just in case A is true at (t,t) iff there is a moment t' earlier than t such that B is true at (t',t'). According to this definition, (17) is not the past tense of (15) since if (17) is true at (t,t), then, evaluating the initial operator P, $F\Lambda x(NPAx \supset Bx)$ would be true at (t',t), for some t' earlier than t, and the N operator here would still refer back to t, the time of utterance of (17). To arrive at the sense of (19), we really want the N operator to refer back to t', the time to which the initial P operator takes us, so that $F\Lambda x(NPAx \supset Bx)$ can be evaluated at (t',t') and thereby fulfil Vlach's condition for the past tense of (15). In order that this should be possible, it is necessary to introduce an operator whose function it is to shift the second member of the pair of moments at which formulae are evaluated, and in particular, to shift it so that, in the present example, it becomes the time to which the initial P operator takes us. Vlach introduces an operator K for this purpose, interpreting formulae of the kind KA along the lines: KA is true at (t,t') iff A is true at (t,t). The formula we are looking for can now be expressed as

(20) $PKF\Lambda x(NPAx \supset Bx)$,

and this formula does satisfy Vlach's condition for the past tense of (15). Note that once Kamp's system is extended in this way, the second member of each pair of moments at which formulae are evaluated is no longer uniformly interpreted as the moment of utterance, so the N operator can no longer be interpreted as a 'now' operator except in contexts where it does not occur within the scope of Vlach's K operator.

3. A Comparison of Kamp's and Vlach's Systems with L

Proceeding along the same lines as the first section of this chapter, let us consider how Kamp's system is related to L by first extending the translation

procedure to cover this extended Priorean language. This is not difficult; we simply add to part II of the procedure the clause

(vi) ${}^{\alpha}NA$ is ${}^{\beta}A$, where β is the moment variable bound by the initial occurrence of Δ in the translation.

According to this procedure, the formula of Kamp's language used to express the form of (7), namely (18), is translated as follows:

(21) $F\Lambda x(NPAx \supset Bx) \rightarrow \Delta^{\alpha\alpha}F^{\beta}\Lambda x({}^{\alpha}P^{\gamma\gamma}Ax \supset {}^{\beta}Bx)$,

and the translation is the same as (8), the formula of L used to express the form of (7). It turns out, however, that not all of Kamp's formulae can be represented by a formula of L in this way (although the adjustment to L which would allow complete translation is simple and straightforward. I shall consider this matter presently.)

As for translation in the converse direction, again if we ignore those formulae of L containing occurrences of Π, Γ, or atomic comparisons, it is still not the case that any formula of the remaining fragment of L is the translation of some formula of Kamp's language. Consider, for example, a formula of the kind

(22) $\Delta^{\alpha\alpha}P^{\beta\beta}F^{\gamma}\Lambda x({}^{\beta}P^{\delta\delta}Ax \supset {}^{\gamma}Bx)$,

which approximates the form of (19). For reasons discussed towards the end of the previous section, Kamp's system is inadequate to express (19), and something at least as strong as Vlach's system is required. The formula of Vlach's system expressing the form of (19) is, as we have seen, (20), of which (22) is the analogue in L. In fact, it seems clear now that whatever Vlach can achieve with his enriched system by way of symbolising statements in English can also be achieved with that same fragment of L where only those primitives occur which were needed for the translation of the Priorean language. We might add to the translation procedure a clause to the effect that ${}^{\alpha}KA$ is translated as ${}^{\alpha}A({}^{\alpha}NB)$, where any subformula NB of A is later translated from the form ${}^{\alpha}NB$, deleting whatever moment variable might otherwise have been attached to the subformula as a result of the translation of A. (To establish this point, however, we still have to consider whether not being able to translate certain formulae of Kamp's system significantly impairs the expressive power of L.)

Putting aside this last reservation for the moment, we can show that the fragment of L in question is even more powerful than the comments so far would lead us to expect. Consider an example similar to (22) but with, amongst other things, a more complex consequent:

(23) $\Delta^{\alpha\alpha}F^{\beta}\Lambda x({}^{\alpha}P^{\gamma\gamma}Ax \supset {}^{\beta}F^{\delta}\Lambda y({}^{\beta}By \supset {}^{\delta}Cxy))$,

which could be used to illustrate the form of

(24) Everyone who has come will be going to meet those who play after the concert,

with Ax standing for 'x comes', By for 'y plays at the concert' and Cxy for 'x meets y'. Trying to formulate (24) with the resources of the extended Priorean languages so far considered, we might begin $F\Lambda x(NPAx \supset F\Lambda y(\ldots))$. But how is the antecedent of the second implication to be formulated? Suppose we try $NPBy$ here, by analogy with the first antecedent. This will not do since we want to go to precisely that time to which the initial F operator takes us; and NBy would be wrong here for similar reasons. So we might try

(25) $FK\Lambda x(NPAx \supset F\Lambda y(NBy \supset Cxy))$,

using Vlach's K operator. But this will not do either. For in correcting the mistake just noted in the second antecedent, the N operator in the first antecedent no longer refers to the present moment, but rather, to the time to which the initial F operator takes us. And again, any juggling of the respective positions of the tense operators and universal quantifiers in (25) will alter the sense.

Thus, even within Vlach's system, propositional tense operators are too cumbersome to match the expressive power of this fragment of L. There is, however, a suggestion in the appendix of Vlach's thesis which perhaps meets the problem. He outlines there a family of stronger systems which contain not just the single K and N operators, but a set of operators $K_1, \ldots, K_n$ together with a corresponding set $N_1, \ldots, N_n$ interpreted in such a way that each operator N_i refers back to a previously established context indicated by an occurrence of K_i when N_i is dominated by (i.e., falls within the scope of) K_i. A system of this kind would circumvent the problems that arose for (25), though in general to be adequate for any tensed context in English a system with denumerably many N and K operators would probably be required. The relegation of this family of systems to an appendix, on the grounds that "the present system [i.e., the one presented in the previous section] was chosen for its relative simplicity, and because it seems sufficient to handle most actual English examples that would naturally be expressed without the use of expressions that refer explicitly to time, like 'the first moment'",[4] hardly seems justifiable. For although statements of the complexity which requires systems of this last kind are relatively rare in English, a systematic logical account of tenses ought to provide a general technique for dealing with all tense constructions; or, at the very least, the frequency with which statements crop up in ordinary usage should not determine the generality of the system.

However, in order to enrich the Priorean language sufficiently to enable it to cope adequately with a reasonable class of tense constructions in English, one of the most convincing aspects of the Priorean analysis of tensed statements is lost. For surely one of the features which has made the Priorean analysis of tenses attractive to logicians is the *prima facie* plausibility of the claim that, since moments

[4] Frank Vlach, op.cit., p. 160.

and dates are not explicitly indicated in ordinary English tense constructions, the underlying statements made by the use of such constructions should not contain moments or dates explicitly either. Evidently, this thesis has been abandoned in advocating the analysis proposed here. I shall return to this theme in the following chapters and evaluate the thesis more fully there. For the present, I would like just to suggest that with the introduction of indexed sets of operators, Vlach's extension of the Priorean language is tantamount to introducing moment variables explicitly into the analysis of tenses. The argument so far has been that, in the first place, Prior's tense operators are too clumsy because they take their entire scope formula to some other time. Kamp's N and Vlach's K allowed a little more selectivity, but this proved not enough. My contention has been that quantifiers, which do not necessarily bind all variables occurring within their scope, possess the minimal properties required of a tense operator to achieve the appropriate level of discrimination within its scope, and associated with the quantifiers we have used an indexed set of moment variables $\alpha_1, \alpha_2, \ldots$ Now it is possible to dispense with the αs, retaining just the indices, so that we have expressions like $\Delta^1, \Pi^{3\,3} F^5$, etc., together with a corresponding set of indices attached to the predicate letters falling within the scope of such expressions. This is all that is required to express patterns of binding for which the usual semantic interpretation can be constructed, and Vlach's paired operators with corresponding indices establish patterns of binding in much the same way. Admittedly, Vlach is able to retain Prior's notion of a proposition which may change its truth value from time to time, and I shall have to take account of this; but it looks (on the evidence of the logical analyses hitherto proposed) as though moment variables, or some proxy for them, must appear explicitly at some point in any complete analysis of tenses.

Apart from these considerations of the adequacy of the expressive power of various logical systems to reflect the principles underlying the behaviour of tenses in English, there is an issue Prior has raised which I feel is rather telling against Kamp's extension of the Priorean language. Prior's point is one we have already noted, namely that he would not readily condone the introduction of two concepts of the present tense (or two present tenses). Since there is already a notion of the present inextricably bound up with the interpretation of tensed statements developed by the use of propositions whose truth values may vary with time in a tense logic, Prior's criticism fell on the 'now' operator. Now, there is in Kamp's system the thesis

(26) $NA \equiv A$,

the validity of which Kamp staunchly upholds (p. 238), which reflects the redundancy of the 'now' operator in contexts where it is the initial tense operator. However, the senses of the two present tenses diverge in other contexts, for whereas the present tense indicated by the 'now' operator refers to the moment of production of the statement in question, the present tense which is understood (so that we need not always read, say, FA as 'It is the case that it will be the case that

it is the case that A') does not do so. Indeed, Prior thought it the essence of presentness that there should be a past present, a future present, and so forth, and that this is what sanctions the use of iterated constructions like 'the future past' and so on, expressed by iterated sequences of tense operators—they are abbreviations for 'the present future present past present', etc.

On the other hand, there is just one notion of the present at issue in the system advocated here, namely that which is indicated explicitly by an occurrence of Δ. When translated into L, (26) becomes $\Delta^{\alpha}({}^{\alpha}A \equiv {}^{\alpha}A)$, which is valid, but certainly does not express the redundancy of Δ in L in favour of some other notion of presentness in any contexts. Perhaps the formula $\Delta^{\alpha\alpha}A \equiv {}^{\alpha}A$ might be regarded more analogous to (26). But this is not valid and so does not express a redundancy of Δ. In fact, it should be clear now that no notion of the present is expressible in L apart from that represented by Δ. The present tense we have thus retained is not the one Prior thought logicians who take tenses seriously must adopt, but this just reflects the substantial difference between our interpretation of tenses and his.

This difference of interpretation highlighted by the discussion of (26) is not just a matter of expressive power of the formal systems of Kamp and mine. For we both agree that 'now' serves to pick out the present moment, which always includes the moment of speaking, and so in this sense any difference of opinion is not so much a disagreement over the meaning of 'now'. Rather, the difference of opinion concerns the centrality of what is expressed by 'now' within tense logic; for on my view there is just one notion of presentness which sometimes has to be emphasised explicitly in English sentences by the word 'now'. A consequence of this divergence of opinion seems to amount to different views on the sort of context in which an operator reflecting the behaviour of 'now' can occur. This is brought out particularly well by another thesis in Kamp's system, namely

(27) $A \supset (NA \,\&\, GNA \,\&\, HNA)$;

and we shall now see why not every formula of Kamp's language can be translated into L.

Note first that G and H are defined in standard Priorean tense logic (and Kamp's system) as, respectively, $\sim F\sim$ and $\sim P\sim$; and in parallel fashion, ${}^{\alpha}G^{\beta}$ and ${}^{\alpha}H^{\beta}$ are equivalent, respectively, to $\sim{}^{\alpha}F^{\beta}\sim$ and $\sim{}^{\alpha}P^{\beta}\sim$ in L, by lemma 7. Accordingly, we can translate GA directly as ${}^{\alpha}G^{\beta\beta}A$ and HA as ${}^{\alpha}H^{\beta\beta}A$ when applying our proceudre to (27). It would appear, then, that the translation of (27) must be

(28) $\Delta^{\alpha}({}^{\alpha}A \supset ({}^{\alpha}A \,\&\, {}^{\alpha}G^{\beta\alpha}A \,\&\, {}^{\alpha}H^{\beta\alpha}A))$.

But (28) is not a formula of L. This is because we do not allow, for any formula A containing no occurrences of a variable v, the string of symbols ΛvA to count as well formed. Of course, this sort of restriction is frequently not imposed on the well formed formulae of first order languages. But this is only as a matter of convenience; usually, no substantial point hangs on the presence of formulae like $\Lambda x(Py \supset Qz)$, and we would be very suspicious of these if this were not the case.

Now, it would obviously be a simple matter to relax the formation rules of L in order to allow (28), and hence to have a translation of all Kamp's formulae in L. But it seems to me that a substantial point is to be made by doing things in the manner adopted here. For statements of the kind '*a* is such-and-such now' are similar to tenseless dated statements of the kind '*a* is so-and-so at *m*' in as much as these statements do not themselves hold true relative to a time. Thus, just as the question 'On what day does Bertrand Russell die on 2nd February 1970?' does not arise, so it strikes me as an extremely unnatural idiom to speak of Bertrand Russell's dying now as occurring at some time, let alone every time. Sentences like 'It has always been the case that *A* now', which are purportedly used to specify the time or times at which statements, themselves expressing the time or times at which something or other holds true, hold true, conflict so much with ordinary usage that their presence in a formal system specifically designed to reflect the idiomatic behaviour of 'now' seems to me to count against the plausibility of that system.

It might be said that this view simply begs the question with respect to Prior's propositions with variable truth values; for if all propositions which we are considering as the subject matter of our logical analysis are like this, then these constructions do arise naturally within the logical sysem. On the other hand, as an analysis of ordinary usage this seems incorrect, and some philosophical argument ought really to be forthcoming to establish why we should adopt propositions with variable truth values, and consequently, the idiom in question. But I am not aware of any convincing argument in support of this view. I shall return to this particular issue in the next chapter; but let me at this point try to make my position clearer by stating that it was precisely to remove any vestige of this unusual idiom, which might otherwise have been thought to linger in L, that formulae preceded by quantifiers binding no variables within their scope were not allowed as well-formed. Moreover, it seems to me that barring these expressions as we have done goes some way towards explaining why the unusual idiom is unnatural.

As an aside from the present line of argument, but in connection with (26) and (27), Kamp holds that both of these ought to be valid on a satisfactory semantics of 'now', and he also maintains that it is impossible to supply a truth condition for statements of the kind *NA*, which satisfies this requirement, of the form

$$\text{the truth value of } NA \text{ at } t, \text{ relative to } \mathbf{M} = \begin{cases} 1 \text{ iff } \ldots \\ 0 \text{ otherwise,} \end{cases}$$

where **M** comprises a set of moments **T**, an order relation on **T** and an assignment of truth values to all atomic formulae at each time $t \varepsilon \mathbf{T}$. He then goes on to develop semantics for the *N* operator with truth conditions of a different form, as we have seen, involving not a single time but a pair of times at which formulae are evaluated. However, I do not think it is impossible to find truth conditions in the form just indicated which do satisfy the requirement that both (26) and (27) be

valid. Suppose $\mathbf{M} = (\mathbf{n},\mathbf{T},<,\mathbf{V})$, where $\mathbf{T}$ is a set (of moments in time), $<$ an order relation on $\mathbf{T}$, $\mathbf{V}$ a valuation function assigning a truth value to each atomic formula at every time $t \varepsilon \mathbf{T}$, and $\mathbf{n} \varepsilon \mathbf{T}$ is a select element of $\mathbf{T}$ interpreted as the present moment. Then the condition

$$\text{the truth value of } NA \text{ at } t, \text{ relative to } \mathbf{M} = \begin{cases} 1 \text{ iff } A \text{ is true at } \mathbf{n}, \text{ relative to } \mathbf{M} \\ 0 \text{ otherwise,} \end{cases}$$

when incorporated with the usual truth conditions for the other operators, and defining validity as truth at $\mathbf{n}$ in every model $\mathbf{M}$, yields amongst the valid formulae both (26) and (27).

4. Quantification over Individuals in Tensed Contexts

Throughout this chapter, where we have considered the relative expressive capacities of various systems, quantification over individuals has always been to the fore. In fact, certain principles relating to the interpretation of such quantification have governed the thread of our argument in earlier sections. Moreover, in the first section we considered the statement 'Some people who have infiltrated the party will try to destroy it' and suggested that the Priorean formula $\forall x(PAx \ \& \ FBx)$ might be adequate here so that we had to look elsewhere to exemplify the deficiencies in the expressive power of the Priorean language. But whether this formula is adequate depends on the interpretation of the quantifier: some suggestions of Prior's on this issue, for example, if adopted might create difficulties. It is appropriate, then, that individual quantifiers should be discussed at this juncture, and that we should make our own views on the interpretation of these quantifiers explicit here in order to complete the discussion on expressive power.

Prior's view of individual quantifiers arose from his notion of the present. On this interpretation, any formula of the kind $\forall xA$ in the Priorean language should be read 'There is an x such that A', where the 'is' here is understood as present tensed: 'It is now the case that there is an x such that A' would be a more perspicuous reading. Just how this interpretation fits in with his overall view of tenses Prior expresses as follows:

> When a quantifier is governed by, say, a tense operator, it is natural to think of it as ranging over such objects as there may be at the time to which the tense operator takes us; for example, 'It will be that something φs' is most naturally read as 'It will be, at some future time, that something *then* existing φs'. On the other hand, a quantifier *preceding* any such operator is naturally taken to be governed by the 'It is the case that ' which is prefixable to everything we say, and therefore to range over what *now* exists. And where these ranges do not coincide—as is bound to be the case where we are considering what is now but once was not. . . —we have to tread carefully.[5]

[5] *Past, Present and Future*, p. 144.

On this note of caution, Prior develops these views on what has become for him a problem of referring to individuals at times when they do not exist in such a way that he maintains "it is arguable that before and after an individual x exists there are no such propositions as φx (though there may be propositions to the effect that there has been or will be *some* individual with such-and-such properties, *viz* those of x)".[6] Prior and others first developed these ideas in a modal logic called Q. He later turned to the problem of axiomatising a Q-like tense-logical analogue of the modal calculus to meet the present issue. But this line of thought has also been developed by Føllesdal[7] from the point of view of semantics, and although he was interested in certain problems he sees in connection with causal modalities, it will be more in keeping with our approach to give a brief exposition of Prior's ideas in terms of Føllesdal's semantic techniques than in terms of his own axiomatic treatment.

Adopting Kripke's techniques, Føllesdal introduces a set of possible worlds, to be interpreted here as moments in time, on which is defined a model **g**. A model is a binary function assigning to each predicate letter P_j^n a set of n-tuples $g(P_j^n,H)$ at each moment H comprising individuals taken from just those which are said to exist at H. The truth value of an atomic formula $P_j^n u_1 \ldots u_n$ at a time H is then defined, relative to an assignment of objects $d_1, \ldots, d_n$, taken from the entire universe of discourse, to $u_1, \ldots, u_n$, as true if $(d_1, \ldots, d_n) \in g(P_j^n,H)$, false if both $(d_1, \ldots, d_n) \notin g(P_j^n,H)$ but each of $d_1, \ldots, d_n$ exist at H, and if $d_1, \ldots, d_n$ do not exist at H, $P_j^n u_1 \ldots u_n$ is without a truth value at H. Truth values are then defined for molecular formulae along the usual lines, but only where the molecular formulae have no components lacking a truth value. In particular, formulae of the kind $\Lambda x A x$ are interpreted in such a way that the quantifier ranges over just those individuals existing in the world at which the truth value of this formula is evaluated.

Prior took these views on quantification and predication, which he considered to arise naturally from the enterprise of tense logic, to have important philosophical ramifications. For example, a consequence of his interpretation of quantifiers in tensed contexts is that the validity of formulae of the kind $FVxAx \supset VxFAx$ is unacceptable, and at one point[8] Prior offers as a reason for accepting the priority of tensed over tenseless discourse the fact that, if we try to define tensed statements in terms of tenseless statements along the lines he suggests, the schema in question comes out valid. Prior's general point here seems to be that there are distinctions to be drawn in tense logic which are naturally and easily formulated there—for example, by Føllesdal's techniques—which preclude the validity of certain schemata, but which are just not available in a logic of tenseless statements. Again, Prior has suggested, as a reason for advocating a tensed in

6. *Papers on Time and Tense*, p. 147.

7. Dagfinn Føllesdal, "A Model Theoretic Approach to Causal Logic", *Det Kgl Norske Videnskapers Selskabs Skrifter* (1966), Nr 2 (Trondheim).

8. *Papers on Time and Tense*, pp. 143–4.

favour of a tenseless language, that we might abolish pseudo-individuals of the kind 'me-at-time t', and so on, if we take advantage of the notion of an individual inherent in his view of quantificational tense logic. Perhaps he had in mind here talk of events as points in Minkowski's four-dimensional space-time continuum, and the thesis that supposedly persisting physical objects are simply collections of ephemeral events of this sort. But it is far from clear that Prior's interpretation of quantification over individuals brings us any nearer a satisfactory account of persisting individuals than does the thesis just mentioned. It could just as well be said that his theories create the same kind of difficulty for the notion of a persisting individual as is supposed to arise for the advocate of a tenseless language. Working with pseudo-individuals like 'me-at-time t' is no more incumbent on the proponent of tenseless language than working with pseudo-individuals of the kind 'the present me' or 'the future me' is on Prior. Their positions are precisely the same in this respect, and I shall maintain that whether or not such fragmentary individuals are adopted is independent of whether we use tensed or tenseless statements. As for his first point, that certain distinctions arising naturally on his view of tenses are not available in a tenseless language, it will transpire that his interpretation of quantification and predication is not at all natural so far as ordinary tensed discourse is concerned. And so, as we pursue our discussion on expressibility, remembering that our criterion of the expressive power of a formal system of tense logic is concerned with statements in English that can be represented by that formal system, we shall see that these arguments of Prior's in favour of a tensed over a tenseless language cannot be sustained.

Consider, then, the statement

(29) The composer who died on 26 September 1945 is now famous,

which we formulate in L by

(30) $\Delta^{\alpha\alpha}A\,Thx\,\Pi^{\beta}({}^{\beta}Cx \;\&\; {}^{\beta}Bx \;\&\; \beta{=}m)$,

where Ax stands for 'x is famous', Bx for 'x dies', Cx for 'x is a composer' and m for '26 September 1945'. Expanding the definite description in (30), and abbreviating ${}^{\beta}Cx \;\&\; {}^{\beta}Bx \;\&\; \beta{=}m$ to ${}^{\beta}Dx$ (for 'x is a composer dying on 26 September 1945,), we get

(31) $Vx\Lambda y((\Pi^{\beta\beta}Dx \equiv x{=}y) \;\&\; \Delta^{\alpha\alpha}Ax)$.

Clearly, then, we have in (29) an example of a property which is said to apply now to an individual who does not now exist, and so there can be no question of the range of quantification being restricted here to what presently exists. Suppose now we try to express (29) along Priorean lines in such a way that the individual quantifiers occur within the scope of a past tense operator, and so, according to Prior's interpretation, range over individuals which existed at some past time.

We might try

(32) $PVx\Lambda y(Dy \equiv y{=}x \;\&\; NAx)$,

availing ourselves of Kamp's 'now' operator, since I do not see how the resources of the original Priorean language would be adequate here. But now in order to provide a semantic interpretation for the subformula Ax as it occurs in the context of (32), predicates must be interpreted in such a way that the extension assigned to them at one time does not necessarily draw on just those individuals existing at that time. According to Føllesdal's semantics, however, (32) would be without a truth value—or unstatable, as Prior would say—since the last conjunct would be without a truth value, whereas (29) is perfectly intelligible and in fact true.

Now, although this example does illustrate the inadequacy of the kind of restrictions formulated in Føllesdal's semantics, it might be suggested that this example is to be catered for in some other way on the basis of Prior's comment that although there are no propositions about x when x does not exist: "there may be propositions to the effect that there has been or will be *some* individual with such-and-such properties, *viz* those of x". Should this be the case, however, the example can easily be modified to serve the same purpose. Instead of (29), consider

(33) Béla Bartók, who died on 26 September 1945, is now famous.

Here the individual quantifiers have gone, but still a property is now ascribed to an individual who no longer exists.

So much for the Q-like restrictions on predicates. As for the restricted interpretation of quantifiers, this has been questioned by a number of logicians who suggest instead that the range of quantification should extend over the entire universe of discourse—over all past, present and future individuals. Richard Montague, for example, has suggested that such unrestricted quantification is necessary to cope with 'There was a man whom no one remembers'.[9] But if we avail ourselves of Kamp's 'now' operator in a Priorean language, this example is not sufficient to establish what is claimed; it only reinforces the point concerning the interpretation of predicates since the example in question can be formulated as $PVx(Mx \,\&\, N{\sim}VyRyx)$, with the existential quantifiers interpreted to range over just those individuals existing at the time to which the immediately preceding tense operator takes us. Dana Scott, however, has produced some more convincing examples involving the comparison of individuals existing at different times; for instance, 'No two presidents of the United States ever looked alike', 'All Nobel prize winners were equally deserving' and 'All readers of the novel will be as deeply moved as I'. Again, it would not be sufficient, now that we have Kamp's N and Vlach's K, simply to produce examples involving comparisons between individuals existing at different times—the examples might only show that Q-like restrictions on the extensions of predicates are inappropriate. But Scott's examples do establish more than this. To illustrate the point, consider his last quoted example, 'All readers of the novel will be as deeply moved as I', which might be formulated with the resources of the Priorean language as $\Lambda xF(Rx \supset Mxa)$, with

[9] Richard Montague, 'Pragmatics and Intensional Logic', *Synthese* 22 (1970), pp. 73–74.

Rx for 'x reads the novel' and Mxa for 'x is as deeply moved as a'. The universal quantifier and the future tense operator must occur in this order, since it is not intended that everyone will read the novel at the same time. But if we were to impose Prior's interpretation of the universal quantifier, it would range over just those individuals presently existing and the formula would clearly not represent the intended sense of the statement in question. I feel that this formula does not do justice to the tensing of the comparison 'is as deeply moved as'. We might try $\Lambda x F(Rx \supset NMxa)$ instead, using Kamp's 'now' operator; but this would not resolve the issue. For it seems clear that two times are involved here: given some individual, say John, who will read the novel and be as deeply moved as I am, there is the time when John becomes as deeply moved as I am, and the time when I am as deeply moved as John will be, and I see no reason for thinking that the relation holds between John and myself at one of these times rather than the other. However, the example is sufficient to indicate the need for an unrestricted interpretation of individual quantifiers.

It would appear, then, that an interpretation of individual quantifiers not restricted in Prior's sense, of the sort we have adopted in Chapter III for L, is appropriate on any view of tenses. A corollary of this view is that Prior's objections to the tense-logical Barcan formula $F\mathrm{V}xAx \supset \mathrm{V}xFAx$ cannot be sustained, and the analogue of this formula in L,

(34) $\Delta^{\alpha}({}^{\alpha}F^{\beta}\mathrm{V}x^{\beta}Ax \supset \mathrm{V}x^{\alpha}F^{\beta\beta}Ax)$,

is valid. (34) simply states that, if at some future time A is true of some individual, then A is true of some individual at some future time, which is quite innocuous. The converse of (34) is also valid. Similarly, the equivalence

(35) $\Delta^{\alpha}({}^{\alpha}G^{\beta}\Lambda x^{\beta}Ax \equiv \Lambda x^{\alpha}G^{\beta\beta}Ax)$

is also valid, and these equivalences also hold when ${}^{\alpha}F^{\beta}$ is everywhere replaced by ${}^{\alpha}P^{\beta}$ and ${}^{\alpha}G^{\beta}$ by ${}^{\alpha}H^{\beta}$. From our point of view, Prior's interpretation of individual quantifiers in tensed contexts can be seen as grounds for imposing restrictions on the interchangeability of two existential or two universal quantifiers when one of these binds individual variables and the other binds moment variables. It is a well established principle of standard first-order logic that a pair of existential quantifiers is interchangeable, and similarly with universal quantifiers, and I am in effect arguing that tensed contexts are an area where this well established principle has application.

More interesting situations arise when either the individual or moment quantifier is existential whilst the other is universal, and again it seems to me that well established principles of first-order logic are at issue here. Consider the following pair of statements:

(36) There are men who will always influence the young.

(37) There will always be men who influence the young.

These two statements are not equivalent since it is compatible with (37) that at different times in the future, different men influence the young, whereas (36) requires that there are men who over the generations will influence the young—Alexander the Great, Jesus Christ or Marx might be considered such men. However, any state of affairs satisfying (36) will satisfy (37), so that (36) implies (37), and this is precisely what we find in L; for whereas

(38) $\Delta^{\alpha}(\mathrm{V}x^{\alpha}G^{\beta\beta}Ax \supset {}^{\alpha}G^{\beta}\mathrm{V}x^{\beta}Ax)$

is valid, its converse is not. This parallels the situation in single-sorted first-order logic where $\mathrm{V}x\Lambda yA \supset \Lambda y\mathrm{V}xA$ is valid, but not its converse.

Similarly, we have as a thesis of L

(39) $\Delta^{\alpha}({}^{\alpha}F^{\beta}\Lambda x^{\beta}Ax \supset \Lambda x^{\alpha}F^{\beta\beta}Ax)$,

whilst the converse of (39) is invalid. To illustrate the point, consider the following pair of statements:

(40) There will come a time at which all particles of matter are disseminated.

(41) All particles of matter will be disseminated.

If we think of these statements in connection with the so-called "heat death" of the universe, (40) suggests that the universe will suddenly attain this state at some future time, whereas (41) allows that there should be a gradual move towards this eventuality. Nevertheless, (40) implies (41) although the converse implication does not hold.

Again, if ${}^{\alpha}F^{\beta}$ is everywhere replaced by ${}^{\alpha}P^{\beta}$, and ${}^{\alpha}G^{\beta}$ by ${}^{\alpha}H^{\beta}$, (38) and (39) remain valid and their converses remain invalid.

Thus far we have found reason for giving individual quantifiers a wide interpretation rather than interpreting them in Prior's restricted sense, and some aspects of the behaviour of unrestricted quantifiers in tensed contexts have been considered. In fact, it has been argued that the behaviour of individual quantifiers in tensed contexts is just one aspect of by now well-established principles of first-order logic governing the interchangeability of quantifiers. Throughout this chapter tacit appeal has been made to these principles, for repeatedly the force of examples has depended upon the order in which tense operators and individual quantifiers occur, where we could rely upon the non-interchangeability of universal and existential quantifiers to maintain the order. It is important to appreciate, however, that this feature has the consequence that the definition of a tense adopted here is not a very convenient one from the formal point of view. The drawback is that since it is not in general possible to reshuffle the quantifiers occurring in the prefix so that members of the chains constituting tenses occur consecutively in the prefix, the implication relations amongst tenses cannot be discussed in isolation from the contexts in which they occur. There are, of course,

analogues of the theses and rules of inference of Priorean tense logic. For example, as things stand it is possible to prove the mirror-image rule to the effect that any valid formula is transformed into another valid formula by systematically interchanging all occurrences of G and F by, respectively, H and P (since the converse of the linear order relation $<$ is also a linear order relation). Analogues of the axioms of Lemmon's minimal system of tense logic K_t are valid in L; for example

(42) $^{\alpha}H^{\beta}(^{\beta}A \supset {}^{\beta}B) \supset (^{\alpha}H^{\beta\beta}A \supset {}^{\alpha}H^{\beta\beta}B)$;

and, provided β is not free in A and γ is free for α in A.

(43) $^{\alpha}P^{\beta\beta}G^{\gamma\gamma}A \supset (\alpha/\gamma)A$

(44) $(\alpha/\gamma)A \supset {}^{\alpha}G^{\beta\beta}P^{\gamma\gamma}A$

are both valid. Moreover, since $<$ is transitive,

(45) $^{\alpha}F^{\beta\beta}F^{\gamma\gamma}A \supset {}^{\alpha}F^{\gamma\gamma}A$

is valid, provided β is not free in A; since $<$ is connected,

(46) $^{\alpha}F^{\beta\beta}P^{\gamma\gamma}A \supset (^{\alpha}P^{\gamma\gamma}A \vee (\alpha/\gamma)A \vee {}^{\alpha}F^{\gamma\gamma}A)$

is valid with the same proviso; if time extends infinitely into the future so that given any moment, there exists a later moment, then

(47) $^{\alpha}G^{\beta\beta}A \supset {}^{\alpha}F^{\beta\beta}A$

is valid; and finally, if time is dense then

(48) $^{\alpha}F^{\gamma\gamma}A \supset {}^{\alpha}F^{\beta\beta}F^{\gamma\gamma}A$

is valid provided β is not free in A. Theses like these may tell us something about certain tenses in certain contexts, but since a tense is not an arbitrary chain of moment quantifiers occurring consecutively in a formula—that is to say, a tense is not the analogue of any consecutive sequence of Priorean tense operators—such theses as these will not lead to implication relations amongst tenses applicable in any context. Perhaps this may be thought inelegant and the definition criticised on these grounds. On the other hand, if the definition is accepted as a fair account of our intuitive notion of a tense, then the fact that the logical properties of tenses are not in general independent of the context in which they occur is itself quite a significant result.

Returning now from this aside to the discussion of the interpretation of quantification over individuals, it does not follow from what has been said that there is no place for something like Prior's restricted quantifiers; only that the use of unrestricted quantifiers is appropriate in the analysis of the usage of tenses in English. Consider the pair of statements

(49) There was a time when everyone was in danger of dying from the plague.

(50) Everyone has been in danger of dying from the plague.

It seems to me that neither of (49) or (50) implies the other, and this because the two occurrences of the word 'everyone' signify possibly distinct sets of individuals. (49) might be paraphrased along the lines 'There was a time when everyone living then was in danger of dying from the plague', and (50) as 'Everyone now living has been in danger of dying from the plague', in order to make the difference explicit.

There are some suggestions in the literature which might be appropriate for dealing with examples like these. Montague[1] and Rescher[2], for example, suggest the introduction of an existence predicate, say S, so that Sx is read 'x exists', and when occurring within the scope of a tense operator, x is said to exist at the time to which the tense operator takes us. I would like to adopt an analysis along these lines, but I have some reservations concerning the interpretation of the predicate S. There is already in the system an existential quantifier which can be used to express that some individual b exists with the familiar formula $\mathrm{V}x(x{=}b)$, and this suggests to my mind that the predicate we want to introduce is used not so much to assert the existence of an individual as to assert when that individual is to be found. All individuals in the domain, as I shall interpret it, have some position in the spatio-temporal framework; but since the system has been developed so far only to deal with positions in time, I propose to fix the interpretation of the predicate letter P^1_3, abbreviated henceforth to S, so that formulae of the kind ${}^{\alpha}Sb$ are read 'b has some spatial position at α'. (49) can now be expressed in the form $\Delta^{\alpha\alpha}P^{\beta}\Lambda x({}^{\beta}Sx \supset {}^{\beta}Ax)$, and (50) in the form $\Delta^{\alpha}\Lambda x^{\alpha}P^{\beta}({}^{\beta}Sx \supset {}^{\beta}Ax)$, and neither of these schemata implies the other.

Nino Cocchiarella[3] has developed another approach to this problem of distinguishing a broad and a narrow interpretation of individual quantifiers in tensed contexts. Each moment is here assigned a domain understood as the domain of actual individuals at the time in question (i.e., those existing at that moment), and each such domain is a subset of the domain of possible individuals which is the same for all moments. There are then two universal quantifiers, one ranging over actual objects and the other ranging over possible objects. The *possibilia* of Cocchiarella's system are said to include all those objects existing at some past, present or future time, but may include more besides, for ". . . the claim that all possible objects actually exist at some time . . . places too heavy a metaphysical burden on tense logic".[4]

1. Richard Montague, 'Pragmatics and Intensional Logic', *Synthese* 22 (1970)
2. Nicholas Rescher, 'On the Logic of Chronological Propositions', *Mind*, 75 (1966).
3. Nino Barnabas Cocchiarella, *Tense Logic: A Study of Temporal Reference* (Ph.D. thesis, University of California, Los Angeles, 1966).
4. Op.cit., p. 63.

Cocchiarella develops his system of tense logic along Priorean lines in as much as the strong analogy with modal logic is maintained. His view of the matter might be considered to arise quite naturally given this analogy, so that a moment in time is regarded in much the same way as a possible world, and in particular the present moment is thought of as we might think of the actual world in modal logic (or even identified with the actual world[5]). But this analogy is not one I wish to adopt, and I am not at all happy with the way Cocchiarella draws the distinction between actual and possible objects, for I do not see why individuals existing at times other than the present should be thought merely possible. Just how the various modal notions combine with those aspects of propositions concerned with temporal reference strikes me as a fascinating topic, and my comments are by no means directed against the enterprise of modal logic. Rather, I am concerned to be clear on what aspect of statements is being analysed, and in this essay our interest lies just with that aspect of propositions concerned with temporal reference. Now, it may well be the case that verb inflexions, particularly those in the subjunctive mood, cannot be completely analysed under this restriction. For example, the analysis proposed here of the 'would -O' form, occurring in statements like 'He said he would come', as involving reference to some point in time later than a moment in the past, might not in general be adequate since modal notions might be relevant here. But until modal notions are explicitly introduced into the syntax of our system, I feel it inappropriate to take half-measures and interpret the domain of objects in such a way that it includes nonexistent *possibilia*. Certainly I see no reason to confound tenses with modalities in the interpretation of a single operator occurring in the syntax. In fact, I feel it places too heavy a metaphysical burden on tense logic to regard objects which exist at some time but are not to be found at present merely as possible objects, whilst those to be found now are to be regarded as actual or real.

A few brief comments on the identity relation will round off this section. According to our interpretation, the identity relation between individuals is not one which holds at a time, whereas in conventional tense logic it is usually assumed that this is not so and the relation can be tensed. One reason why this policy has been followed in tense logic along Priorean lines is that in the semantics, the interpretation of individual constants is relativised to a time, and there is the possibility that individual constants denote different entities at different times. Now, we have found no reason to relativise the interpretation of constants in this way, for my interpretation of individual quantifiers and of comparisons does not require this; but there has been a suggestion of why we should not relativise in this way. Prior has expressed dissatisfaction with pseudo individuals of the kind 'me-at-time t', and we have seen that simply changing the expression 'time t' to 'the present' or some such hardly suffices to remove the problem. What seems to be pseudo about these individuals is their relativity to a time, and it is precisely this that is avoided in our

[5] Prior expresses this view in 'The Notion of the Present', *Studium Generale*, 23 (1970).

treatment. To say that one and the same individual persists through time so that we can say this individual had such and such properties and now has such and such other properties is to say that we can name this individual in such a way that the name does not refer correctly at one time but not at another: the reference of the name is independent of time. In view of this, I do not see why the identity relation holding between such names should be thought to hold at some times and not at others. Moreover, the identity relation holding between terms referring to individuals is surely the same relation as the identity relation holding between terms referring to moments; but it is never suggested that identity in the latter case holds relative to a time, and so why should it in the former case? Since it is the same relation at issue in both these cases, a satisfactory argument to show that identity between individuals holds relative to a time must also show that identity between moments also holds relative to a time; but I see no prospect of success in this respect.

E.J. Lemmon has argued in favour of relativising identity statements to a time in reply to a paper of Donald Davidson's,[6] though his discussion involves only terms referring to individuals with no mention of terms referring to moments; but apart from the objection just mentioned his proposal still seems to me unconvincing. He suggests (pp. 97–98) that

(51) I flew my spaceship to the Morning Star

does not entail

(52) I flew my spaceship to the Evening Star

in spite of the identity

(53) The Morning Star is the Evening Star.

Rather, what is required, he suggests, to obtain (52) from (51) is more like 'The Morning Star was at that time the Evening Star'. Now, it is difficult enough, so it has been argued,[7] to understand how the Morning Star could not be the Evening Star in some possible world given that the Morning Star is in fact the Evening Star, and Lemmon does not go on to explain and make intelligible the possibility of the Morning Star not being the Evening Star at some time, even though it was the Evening Star at the time in question. Yet if this is not a possibility (and it seems to me it is not), there seems to be little sense in relativising the identity between the Morning and Evening Stars to any time. Instead he gives another example where the supposed referential opacity of tenses is more plausible. (51), in conjunction with the identity statement

[6] E.J. Lemmon, 'Comments on "The Logical Form of Action Sentences" by Donald Davidson' in N. Rescher (ed.), *The Logic of Decision and Action* (Pittsburgh, 1966).

[7] S.A. Kripke, 'Naming and Necessity' in Harman and Davidson (eds.), *Semantics of Natural Language*.

(54) The Morning Star is the first planet struck by a man-made missile, is said not to entail

(55) I flew my spaceship to the first planet struck by a man-made missile

(though they may entail 'I flew my spaceship to what was later to be the first planet struck by a man-made missile', assuming the date of the flight precedes the date of the landing of the Russian missile). Lemmon proposes that "only one principle seems quite safe: if $F(a)$ is true at time t, and at that same time $a=b$, then $F(b)$ is true at time t" (p. 98), and presumably he supposes that at the time at which (55) is said to be true (before the landing of the Russian missile), (54) is not true. But all this example points to is that at that time we might not have known that (54) is true—we could not be certain since the missile had not landed then. It does not follow that the truth of (54) is relative to a time even if our knowledge of it is, yet Lemmon's argument seems to depend upon this.

Finally, Lemmon challenges us to try

(56) What was Gaul is now France

in

(57) Caesar conquered Gaul

and

(58) France is a Republic,

and I see no real problem here. (56) might be paraphrased 'What was called Gaul is now called France', and expressed in the form

(59) $Thx\Pi^{\alpha\alpha}Ax = Thy\Delta^{\alpha\alpha}By$

with Ax standing for 'x is called Gaul' and By for 'y is called France'. (57) can then be paraphrased as 'Caesar conquered what was called Gaul' and expressed in the form

(60) $\Pi^{\alpha\alpha}CaThx^{\alpha}Ax$,

with Cax for 'Caesar conquers x', from which we can derive, together with (59)

(61) $\Pi^{\alpha\alpha}CaThy\Delta^{\beta\beta}By$,

read as 'Caesar conquered what is now called France'. Substitution into (58) follows along similar lines, giving 'What was called Gaul is (now) a Republic'. Alternatively, we might take the view that 'France' is a proper name, which we can represent by 'b', and (56) can be expressed in the form

(62) $b = Thx\Pi^{\alpha\alpha}Ax$.

If (58) is then expressed in the form $\Delta^{\alpha\alpha}Db$, where Dx stands for 'x is a Republic', substitution of identities yields $\Delta^{\alpha\alpha}DTh\Pi^{\beta\beta}Ax$, i.e., 'What used to be called Gaul is now a Republic'.

Chapter VI

Dates and Temporal Order Relations in Tensed Contexts

1. Order Relations and Dates in L

As the themes of Chapter I have so far been developed, the emphasis has been mainly on the analysis of tensed statements. In this chapter I want to bring tenseless statements involving dates and the 'earlier than' relation more firmly into the picture. It is clear, I hope, that one of my aims has been to be able to express statements involving dates and the temporal order relations together with tensed statements so that we have some account of the relation between statements of the kind considered in the first part of McTaggart's argument. Bearing this issue in mind, I shall go on to compare the account of dates, the temporal order relations and tenseless statements, and their relation to tenses and tensed statements, that we find in L, with other systems that have been proposed by various authors which bear on these matters.

In Chapter III of *Past, Present and Future* Prior discusses the correlation between certain properties of the 'earlier than' relation appearing in contemporary accounts of the semantics of his tense operators with various axioms governing the behaviour of these operators. In order to illustrate what Prior has in mind here, consider the assumption of the denseness of time: for any moment t_1 earlier than t_2 there is a moment t_3 between t_1 and t_2. If this condition is imposed in the semantics, the formulae $F \supset FFp$ and $P \supset PPp$, which appear as axioms in some systems of tense logic, are valid, but not otherwise. Accordingly, these postulates are said to express the denseness of time within the object language. Now, Prior continually resorted to the order relations amongst moments of time to explain the intuitive content of formulae like $F \supset FFp$ which are correlated with a specific property of the order relation. To give just one example, he says "if time were discrete, then it could be said that something will be the case for the last time in the moment that is *just* to come; there will then be no moment at which it will be the case *that* it will be the case (the two 'wills' take us to at least two moments hence, and by that time, *ex hypothesi*, p is true no more)."[1] But in spite of this he adamantly maintains that his tense operators are prior to the order relations. Thus, speaking of the correlations between his axioms and the properties of the 'earlier than' relation, he says:

[1] *Past, Present and Future*, p. 39.

> These correlations . . . were suggested by the Russellian method of eliminating tenses, but they were not intended to serve the same end . . . Metaphysically, a translation the other way round would be desirable . . . and as a first step we may point out that 'The date of *p*'s occurrence is later than the date of *q*'s occurrence' seems to be equivalent to 'It either is or has been or will be the case that it both is the case that *p* and is not but has been the case that *q*' . . . The negative part of this is perhaps not necessary; without it, the formula is a variation on McTaggart's definition of 'earlier'.[2]

Whether "metaphysically, a translation the other way round is desirable" is open to question, and in any case I very much doubt that it is possible. One possible motive for reduction of the 'earlier than' relation along these lines, to do with the relative factual content of tensed and tenseless statements, has already been considered and rejected in Chapter I. Apart from the considerations of McTaggart's views in the first chapter, which were not well-received there, I do not know why the reduction should be thought desirable. But leaving questions of motivation aside, I cannot see how any construction from devices which indicate only pastness, presentness and futurity, and do not presuppose in addition the order relations 'earlier than' and 'simultaneous with', will suffice to yield the ordered structure associated with moments in time. To distinguish moments as to presentness and pastness, for example, tells us nothing about the structure of the past; yet moments earlier than the present which constitute the totality of the past form a linearly ordered set. Much the same point is expressed in Chapter IV of Richard Gale's *The Language of Time*. He suggests there that in any attempt to analyse the 'earlier than' relation in terms of the expressions 'past', 'present' and 'future' it is necessary to introduce the relations '*X* is more past than *Y*' and '*X* is more future than *Y*'. But he is unable to refute the obvious charge of circularity according to which these latter two relations are just synonyms of something like '*X* is past and *Y* is earlier than *X*' and '*X* is future and earlier than *Y*', respectively.

It might be retorted that Prior has available constructions like $A \,\&\, PB$ which, when falling within the scope of a past tense operator, for example, might be said to render a counterpart of the ordered structure of the past. But it is possible to give a counterpart of that ordered structure only because Prior's so-called tense operators do not indicate distinctions between past, present and future (it is not possible to iterate these terms); rather, they are just paraphrases of statements involving the temporal order relations. In fact, the past tense operator *P* would appear not even as strong as Gale's relation '*X* is more past than *Y*', which means '*X* is past and *Y* is earlier than *X*'. For it is possible that the *P* operator occurs within the scope of an *F* operator, in which case the *P* operator might not actually take us to a moment in the past, but merely to one earlier than that to which the *F* operator takes us. So the *P* operator does not indicate pastness, and similarly the *F* operator does not indicate futurity.

My reason for claiming that Prior's tense operators are just paraphrases or syn-

[2] Ibid., p. 41.

onyms of constructions involving the temporal order relations is that it is through the semantics that an interpretation of his tense operators is provided, and the order relations appear there in the definitions of satisfiability and validity in terms of which the criteria of soundness and completeness are formulated. Moreover, according to the views developed here, tenses serve to relate events via the temporal order relations to a reference point established in the context of use. The distinctions of past, present and future indicated by the reference points remain fixed in all subordinate contexts so that in different clauses, and even throughout different sentences, various temporal relations may be expressed with respect to a single reference point. The indexical quantifiers were introduced to serve the former function of marking the distinctions between past, present and future, whilst the expressions ${}^{\alpha}F^{\beta}$, ${}^{\alpha}P^{\beta}$, and so on, serve only to order times, specified by existential or universal quantification, with respect to the reference points. Now, as we have seen, Prior's tense operators are analogues of the expressions ${}^{\alpha}F^{\beta}$, ${}^{\alpha}P^{\beta}$, and so on, which were defined explicitly in terms of the 'earlier than' relation and standard quantifiers, but not the indexical quantifiers.

The import of Prior's tense operators is obscured by the fact that his own treatments of tense logic were always from the axiomatic point of view, and the 'earlier than' relation was usually suppressed. In the more recent semantic developments, on the other hand, the issue remains somewhat confused because tenses and the order relations are separated by the distinction between object language and metalanguage. One reason for choosing first-order logic in the form adopted here was so that this particular issue could be more clearly presented in virtue of the fact that the order relations and dates appear alongside expressions indicating pastness, presentness and futurity. Having selected the object language in this way, it was possible to show with our definition of a tense how the order relations are involved in what is expressed by linguistic tenses, and to locate the vagueness in the reference to times that is sometimes associated with the use of tenses in existential and universal quantification. It follows from this view of tenses that there can be no question of eliminating the 'earlier than' relation and those tenseless statements in which it occurs in favour of tensed statements.

As for tenseless statements involving dates and their relation to tenses and tensed statements, we argued in Chapter I for a conditional equivalence of the kind 'If it is now m, then X occurs at m if and only if X occurs now'. This can be generalised somewhat and represented by the schema.

$$(1) \quad \mathbf{W}^{\alpha}(\alpha{=}m \supset ({}^{\alpha}A \equiv (m/\alpha)A)),$$

where **W** stands as usual for any indexical quantifier, and (1) is valid.

Some attempts have in fact been made by Prior and others who adopt much the same view of tenses as he does to introduce dates and the 'earlier than' relation into the object language alongside tense constructions and to investigate their relation to one another. I shall go on to consider how well Priorean tense logic can accommodate explicit mention of dates and the temporal order relations in the

following sections. Finally, in the closing chapter I shall summarise my arguments against tense logic in the style of Prior and his followers.

2. Prior's Account of the Relation between Tenses, Dates, and the Temporal Order Relations

In his article 'Tensed Propositions as Predicates'[3] Prior points to the *prima facie* distinction between propositions like 'Socrates was sitting down' of which it makes sense, so he suggests, to ask when they are true, and propositions like 'Two plus two is four' and 'The date of the Battle of Hastings is 1066' of which it does not so obviously make sense to ask this question. Propositions of the first sort he calls 'tensed', and propositions of the latter sort he calls 'untensed'. I shall continue to call propositions of the latter kind 'tenseless' for the sake of uniformity: it is pretty clear that the two terms are intended to refer to much the same class of propositions. It is Prior's view that this distinction is only a *prima facie* one, for he says

> Our view of the relation between the two sorts of proposition is that 'untensed' propositions are improperly so called; they are simply tensed propositions which are so obviously either always true or never true that to ask *when* they are true is socially (but not logically) inept. The most plausible alternative to this view is to say that tensed propositions are not genuine propositions at all but predicates of instants ('Socrates is sitting down' really means 'Socrates is sitting down at _'), the instants which form their subjects being unmentioned, but usually understood to be the time of utterance.
>
> Of these two views about the relations between tensed and untensed propositions, I am myself a determined adherent of the first, i.e., the view that so-called tenseless propositions are simply a sub class of those tensed propositions which are always or never true. (TPP, p. 290)

Whether the second of these views is in fact the most plausible alternative to the view Prior advocates is something I would dispute; it is certainly not the view implicit in the way L has been constructed. This will become apparent, if it is not already so, as we discuss the series of formal systems constructed in Prior's paper 'Tense Logic and the Logic of Earlier and Later'[4] and elaborated in TPP, where he gives some formal expression of the two views mentioned in the passage just quoted, gradually developing the position he maintains as we come to the last of these systems. In his system I and I+ a syntactical distinction between tensed and tenseless statements is maintained, but this is lost in system II where we find only tensed propositions. Then in system III (which becomes a family of four systems in TPP; but we need not enter into this here) moments are themselves regarded as

[3] *American Philosophical Quarterly*, 6 (1969). Henceforth, this article is referred to in the body of the text as TPP.

[4] Ch. XI of *Papers on Time and Tense*. Henceforth, this article is referred to in the body of the text as TEL.

tensed propositions true at that instant only, and at this point Prior has available an equivalence which will serve as a basis for a definition of 'earlier than' in terms of tensed propositions. Dated propositions are finally defined in terms of his tense operators in system IV.

Now, as Prior progresses from one to another of these systems, becoming more "tense-logically involved" as he puts it, he makes what seem to me implausible assumptions at several stages. Indeed, I do not think he represents the views of those who would oppose him in a satisfactory way in the first of these systems, which is the foundation on which he builds. I shall endeavour to point out these features in a brief survey of the systems.

With a vocabulary that is for the most part familiar by now (I have adjusted the Polish notation and one or two other signs to conform with the notation used here), the tensed propositions of system I are defined by the rules:

(i) The propositional variables $p,q,r,$ etc., are tensed propositions;

(ii) If A and B are tensed propositions, so are $A \supset B$, $\sim A$, GA and HA.

F and P are defined as $\sim G\sim$ and $\sim H\sim$, respectively, and the other propositional connectives as usual. Tenseless propositions are defined by the rules:

(iii) If u and v are moment variables, $u=v$ and Euv are tenseless propoistions;
(iv) If u is a moment variable and A a tensed proposition, TuA is a tenseless proposition;
(v) If A and B are tenseless propositions and u is a moment variable, $A \supset B$, $\sim A$ and ΛuA are tenseless propositions.

Formulae of the kind TuA are read 'A is true at u' or 'It is the case at u that A'. The postulates of system I over and above an adequate basis for standard quantification theory are:

$ET1$ $\quad Tu(p \supset q) \equiv (Tup \supset Tuq)$
$ET2$ $\quad Tu{\sim}p \equiv {\sim}Tup$
$UT1$ $\quad TuGp \equiv \Lambda v(Euv \supset Tvp)$
$UT2$ $\quad TuHp \equiv \Lambda v(Evu \supset Tvp)$.

The symbol T is superfluous in this system since TuA could be written as Au and the axioms could then be regarded as defining complex predicates involving propositional connectives (see TPP, p. 291, for details). Moreover, G and H can be eliminated from the primitive terminology by construing $UT1$ and $UT2$ as definitions, and in that case we might say that tense logic is to be seen as a by-product of the logic of the 'earlier than' relation. Thus, system I would appear to be a formulation of a position opposed to that which Prior maintains. Even so, it is not the position advocated here in opposition to Prior. On our view, it could be said that there are two ways of making temporal reference definite: either by using

a tense or by some other means (by using a date or by using existential or universal quantification over moments). Consider, for the sake of illustration, a simple formula of the form ${}^{\alpha}P^{1}_{j}u$. As it stands, this formula is neither tensed nor tenseless; its time reference is indeterminate because of the occurrence of a free moment variable. If the moment variable were to be bound by a chain, however, the resulting expression would be said to be a tensed statement. On the other hand, if the moment variable were bound by just a universal or an existential quantifier, or was replaced by a date, the resulting expression would constitute a tenseless statement according to our definition. It is definitely not the case on this view that a tensed statement is thought of as incomplete with respect to the aspect of temporal reference, and as far as this goes we are in agreement with Prior that tensed statements are just as much "genuine" propositions as are tenseless propositions (perhaps we should say "*prima facie* tenseless propositions" in expressing Prior's view). But on our account tenseless dated statements are not formed by modifying a tensed statement in the manner suggested in Prior's formation rule (iv), namely by prefixing an operator read as 'It is the case at u that' to an already tensed statement. The reason I think this view of dated statements is inappropriate is similar to that raised in criticism of Kamp's 'now' operator. For just like 'It always has been the case that A now', it appears that 'It is now A at six o'clock' would be used in an attempt to specify the time (i.e., six o'clock) at which a statement, itself expressing the time (i.e., now) at which something holds true, holds true. Perhaps the possibility of being able to formulate this idiom within a consistent formal system means that it is not nonsense since some interpretation can be found. But as an analysis of English usage it is just wrong.

In system I^{+} the class of tensed propositions is enlarged so that if A is a tensed proposition, B is tenseless and u a moment variable, then $A \supset B$, $B \supset A$ and ΛuA are tensed propositions. Some further postulates are laid down, but these need not be considered here; they become redundant in system II. The distinction between tensed and tenseless propositions still remains in this system, but not so in system II. Here the tenseless propositions of systems I and I^{+} are counted as tensed and expressions of the kind $TuTvA$ are well formed, where u and v are moment variables. Three more axioms are laid down which equate these formulae which were not well-formed in the former systems with formulae that are well-formed there:

$ET3$ $\quad TuTvp \equiv Tvp$

$ET4$ $\quad TuAvTvp \equiv AvTvp$

$ET5$ $\quad TuEv_1v_2 \equiv Ev_1v_2$

There is also an additional rule of inference RET to infer A from TuA when u is not free in A, and vice versa.

Now, these new formulae seem even less plausible than those got by construing tensed propositions as predicates of instants. My criticism is of much the same kind, namely that attempting to give the time at which something holds true at a

time is not very easily intelligible and is certainly an artificial idiom, but the problem is more explicit now since both times are specified in the same way in formulae like $TuTvA$. The situation is slightly different with formulae of the kind $TuEv_1v_2$. Here it seems we have the same mistake as that attributed to McTaggart in Chapter I, for there is no time at which one moment is earlier than another and it makes no sense to ask for such a time. It will not do to say with Prior that the initial Tu is vacuous in formulae like $TuTvA$ and $TuEv_1v_2$ since we want to say that it does not make sense to prefix an operator to a sentence like 'The Battle of Hastings occurs in 1066' in an attempt to specify when it could be true, and any reasonable analysis of such expressions should reflect this feature in both the construction of wffs and their logical properties.

System III is heralded by another startling assumption. Moments are here regarded as propositions to be included within the class of tensed propositions, and the additional axiom

$$ET6 \qquad Tuv \equiv u{=}v$$

is introduced, with the identity relation construed now in such a way that it can hold between propositions. Prior himself remarks that this is a highly artificial procedure. But "what lies behind it is the belief that 'instants' are artificial entities anyhow, i.e., that talk which appears to be about them, and about the 'time series' which they are supposed to constitute, is just disguised talk about what is and has been and will be the case" (TEL, p. 123).

This is not a thesis I wish to maintain, and so the procedure according to which moments are regarded as propositions remains as artificial and implausible as ever. I think Prior found it easier to maintain a thesis of this kind than perhaps some other tense-logicians do since he never developed his systems from the semantic point of view, in which case moments and order relations would appear in the metalanguage. Having strayed so far from what is intuitive and idiomatic, however, Prior is able to establish within his system III an equivalence between Euv and $TuFv$ ('It is true at u that it will be the case that v'), and on the basis of this he was able to eliminate the 'earlier than' relation from his primitive terminology. Finally, Prior completes his program with his system IV where it is possible to eliminate T from the primitive terminology in favour of his tense operators. Thus, having begun with system I where G and H are defined in terms of the primitives T and E, he is able in system IV to regard his logic of E and T as a by-product of his tense logic.

This, in outline, is how Prior carries out his reductionist program: he develops a system which suits his metaphysical predilections at the expense of providing an analysis of English usage. It is difficult to see why anyone who is not already persuaded of the advisability of eliminating direct reference to moments and their order relations in favour of tensed statements should be convinced by the program. For no arguments are given for the plausibility of assumptions, which, for me at least, are certainly not obvious or appeal to the intuition, independently

of the reductionist program itself. Moreover, apart from the nature of the assumptions to which we have drawn attention in this section leading up to the equivalence between *Euv* and *TuFv*, it is not so easy to see the force of the proposed reduction of *Euv* to *TuFv* once the semantics of Priorean tense operators are developed. Of course, Prior's intended reading of *TuFv* is '*v* is future at *u*'; but as already indicated, I do not think the semantic analysis of the *F* operator bears him out on this point. The phrase 'is future at' occurring here is just a synonym for 'is later than'.

3. Mott's Account of Dated Statements

Bearing in mind the issue arising from the first part of McTaggart's argument, I take it that it is a reasonable criterion of adequacy to impose on any formal system for which it is claimed that it captures our intuitions on temporal reference that it should be able to accommodate both tensed and dated statements in a natural way. Though Prior shared our interest in this issue, he seems, however, to have been too much concerned with his own reductionist program to provide a reasonable analysis of sentences in which mention of times and the temporal order relations are more explicit than in other cases. The question still remains, then, of whether Priorean tense logic can be said to accommodate statements of this kind.

P. Mott[5] has proposed a more sober account of dated statements within the Priorean framework. No attempt is made to eliminate those constructions involving dates which appear in the object language of his system. But the 'earlier than' relation does not appear in his object language, so we should not expect anything like a complete answer to the problem of articulating the relation between tensed and tenseless statements. A relation appears in the semantics, providing an interpretation in the usual way for the primitive operators *G* and *H* for which axioms are laid down amounting to Lemmon's minimal system K_t, and accordingly no conditions are imposed on this relation. Dated statements are formulated with an operator *T* read as 'It is the case on _ that _', in line with Prior's suggestion, and the logic proposed for this operator is much the same as that formulated in Prior's system II. *ET*1, *ET*2 and *ET*3 are in fact theses; *ET*2 is laid down as an axiom together with the implication from left to right in *ET*1 and from right to left in *ET*3, the converses not being independent within Mott's system. Finally, there are two 'mixing' axioms:

GT	$TuA \supset GTuA$
HT	$TuA \supset HTuA$

Dates are formulated here as constants, and *u* and *v* should be thought of as such in these axioms. Moment variables do not appear.

[5] P. Mott, 'Dates, Tenseless Verbs and Token Reflexivity', *Mind*, 82 (1973).

There were two criticisms of Prior's system II made in the previous section which also apply to Mott's system. First, it seems to me unnatural to try to express at what time a tensed statement, say a past tense statement, holds true by means of a formula of the kind *TuPA*. For if *PA* were a proper past tense statement taking us to a moment in the past, it would express that some state of affairs obtains or some event occurs earlier than the present moment, and it does not make sense to say that there is some date at which this state of affairs' obtaining earlier than the present moment holds true, or that there is some time at which this event's occurring earlier than the present moment holds true. This follows from the view of tenses expressed in Chapter I and developed in succeeding chapters. Second, it is even more obviously wrong to ask at what time does a dated statement hold true, and consequently a system containing formulae of the kind *TuTvA* is inaccurate in at least this respect in providing an analysis of the use of dates in English. Formulae of the kind *GTuA* and *HTuA* are suspect for the same reason.

What is interesting about Mott's system, however, is that he proposes a semantic analysis which reflects his view of the status of tenseless statements and in terms of which completeness and soundness of his system can be established. In particular, the satisfiability clause governing the interpretation of the operator *T* runs along the lines: *TuA* is true at *t* relative to a model **M** iff *A* is true at the time assigned to *u*, relative to **M**. It is easy to see why, as a consequence of this rule, formulae of the kind *ET*3, *GT* and *HT* are valid. The time at which a formula *TuA* is evaluated is irrelevant and disappears on the right hand side of the satisfiability clause, so any operator which has the function of indicating the time when subsequent formulae appearing within its scope are true can appear in front of *TuA* to yield an equivalent formula. (This is not true under all conditions. If time has a first or a last moment and *TuA* is evaluated at either of these times, it would not follow that *PTuA* or *FTuA*, respectively, is true at that moment, although *HTuA* and *GTuA* would be. But the possibility of time's being bounded in this way does not substantially affect the issue.)

The motivation behind this rule is Mott's dislike of tenseless verbs, and so dated statements are paraphrased in the form 'It is the case at *u* that *A*' with the bogus present tense 'the present at *u*', similar to Prior's 'future present' and 'past present'. Mott makes the point that tenselessness is not a simple grammatical feature of a verb, so that a simple grammatical test will not do to distinguish tensed and tenseless verbs. Then he suggests that the notion of a tenseless verb is obscure. He represents a dated sentence in the form 'D . . . v . . .', where 'D' is understood as a date modifying a verb 'v', and goes on to ask "what is the special way of understanding the verb, different from what we would adopt in ' . . .v . . .', but necessary when once a date is added?"[6] In the face of this question, he offers two considerations against introducing tenseless verbs: first, it would introduce a

[6] P. Mott, 'Dates, Tenseless Verbs and Token-Reflexivity', *Mind*, 82 (1973), p. 75.

semantic distinction which has no grammatical counterpart; and second, he suggests that a semantics of tenses and dates analogous to the semantics of necessity would be a gain at least in simplicity.

Taking this second point first, and very briefly, it seems to me it has yet to be shown in what way tenses are analogous to modalities, for the concepts of past, present and future cannot be iterated as it is usual to iterate modalities. Moreover, our considerations of quantification over individuals and the extensions of Vlach's system necessary to cope in this area do not suggest that appeals to simplicity are appropriate to justify making the analogy. Turning now to the first point, let me remark first of all that we might well take the view Prior outlined as the "most plausible alternative" to his own view, and which he tried to capture in his system I outlined in the previous section, to constitute one way of answering the question Mott raises. But as we saw in the last section when considering system I, this question need not arise. For it is not necessary to construe dated statements as statements formed by modifying a tensed statement. This position depends, of course, on not following surface grammatical form exactly, but rather, using one's intuition on what the grammatical forms at issue here are or could be used to express. As we have seen in Chapter II, there is good reason to believe this to be necessary in the analysis of tenses, considerations of dates aside. For the same linguistic tense (at any rate, if this is understood as a verb inflexion together, in some cases, with auxiliary verbs) is not always used to express the same structure of the kind characterised here as a tense. If this is indeed so, then it does not seem so much out of keeping with the policy adopted elsewhere in the analysis that a distinction should be introduced which may not be marked in an immediately obvious way in the surface grammar but which is not otherwise mysterious or hard to discern.

Moreover, the point can be made against Mott that although he claims to be following grammatical structure, it is by no means clear that he is in fact doing this. I have already made some criticisms regarding the peculiar behaviour, relative to the usual idioms, of constructions involving dates in various contexts formulated in Mott's system, and my reservations on the affinity of Priorean tenses with ordinary usage also carry across to the corresponding parts of Mott's system. However, Mott does point to a familiar sort of context in support of his views where his analysis does appear, *prima facie*, to be more readily applicable, and it will be fruitful to consider this kind of context.

The situations Mott envisages have in common the feature that there is a significant delay between the time the speaker or writer constructs his sentences and the time the listener or reader with whom he is communicating receives the report. For example, I might have written a letter to a friend, dated 18th June 1970, and used the sentence 'It is raining' to convey that it was then raining. Now, if on the following day when my friend received the letter the weather was fine and it was no longer raining, he would not have called me a liar just because he could no longer express a true proposition using the sentence 'It is raining'. On the strength

of this, Mott maintains (p. 77) that "there is no reason that I can see why the present tense 'It is raining' being true on the 18th June requires that it *is* at present the 18th June." For, with respect to the example, he says that my friend "sees the date at the top of the letter and understands my words as being of the 18th" (p. 77). From a consideration of contexts of this sort, Mott proposes that dated statements in general be analysed in terms of constructions of the kind ' "It is raining" is true on 18th June 1970', with this notion of 'the present at *u*', where *u* is a date.

It seems to me, on the other hand, that contexts of this sort are special cases, even though they are familiar, and certainly do not justify the introduction of the concept of 'the present at *u*' for all contexts involving dates. I shall propose an alternative view of these situations. The special feature of these contexts is the delay involved in the communication. Ordinarily, when people communicate in close proximity the small delays involved are not significant so that, as Rankin pointed out in the passage quoted from him in the last section of Chapter I, the speaker offers his token for purposes of identifying the times indicated by the use of tense constructions, though he does not necessarily refer to it. (Sometimes this is not enough and some times other than the present moment must be established in the context so that there is a point of reference in the past or the future. But we can leave this complication aside in the present argument.) Accordingly, someone might have said to me 'I will be in London when the festival begins', and I can now report to a third party what was said to me (as distinct from what sentence was used) by using the sentence 'He said he would be in London now', the festival having begun. That is to say, that a state of affairs (a certain man's being in London at a certain time) obtains, is expressed at different times by using different sentences, and in particular, different linguistic tenses, to take account of the change in the speaker's point of view. But in both cases the utterances serve to locate the reference point (the present moment). The situation is not the same when I write in a letter that it is raining, because the token on the paper will not itself serve to locate the time indicated by the tense. This is because written tokens and tokens on a tape recorder persist, whereas spoken ones do not, and unless my reader is present to observe me as I write, my written tokens will not serve to identify a moment unambiguously. Consequently, it is necessary to make explicit the date on which I write, say 18th June 1970, so that what I wanted to say about the rain could then be concisely expressed by using the sentence 'It is 18th June 1970 and it is raining'. The important point to notice now is that in order to report to a third party the state of affairs I communicated in the letter, my friend (having received the letter on the 19th) would use the sentence 'It rained on the 18th June', with a past tense inflexion of the verb. He certainly would not use the sentence 'It is raining on the 18th June' since it would not have been true to say 'It is the 18th June' then. According to Mott's account of dates, however, he could say this. Indeed, on Mott's account, to say 'It was raining on the 18th June' would necessarily imply that it rained (rains, will rain?) earlier than the 18th June.

It is extremely doubtful, then, that Mott is in fact following grammatical form. He introduces a construction for dated sentences which imitates surface grammatical form in a few restricted contexts but which is held to be generally applicable to sentences involving dates, and in so doing introduces a semantical concept 'the present at *u*', where *u* is a date, which is just not part of the usual way of thinking about time as this is reflected in the structure and use of the language.

I should add, to allay possible misunderstandings, that I do not claim that the theory advocated here gives an explicit account of the way a state of affairs is reported at different times by the use of different sentences. That is to say, I have not proposed a theory of how a sentence used appropriately at one time can be converted into a sentence which could be used appropriately at another time to report the same state of affairs (though I shall make some suggestions along these lines later). Perhaps those who follow in the Priorean tradition adopt the notion of a proposition (or whatever the wffs of the formal language are intended to represent) which may change its truth value from time to time in an attempt to provide a theory of this kind. However, whereas the account of tenses advocated here does not go as far as this (I shall only be able to sketch in a rough way the lines along which the problem might be solved), Priorean tense logic, at least with respect to the account of dates we have examined, appears to preclude the possibility of a solution and to misrepresent the situation. I shall suggest in the next and final chapter that also with respect to tenses, leaving dates aside, the notion of a proposition with variable truth value is misleading.

Chapter VII

The Argument against Priorean Tense Logic

It will be convenient to summarise my reasons for rejecting the Priorean analysis of temporal reference here and indicate how the points made in the previous chapter fit in with the overall pattern of argument. We began with the proposal that iterated constructions of the kind 'the future past', 'the past present', and so on, which Prior adopted from McTaggart, have no natural counterparts in the use of tenses in English. Rather, it was suggested that the principle that the point of view of the speaker dominates all subordinate contexts, which Prior explicitly rejects at the outset as an incidental and unimportant feature of idiomatic usage, leads to a more natural analysis. This latter view of tenses was supported against the Priorean account on two main grounds: (i) the evaluation of the truth of all statements relative to a single time is inadequate; and (ii) the behaviour of quantifiers ranging over individuals in tensed contexts supports the view that the division of time into past, present and future remains the same in all subordinate clauses and suggests that tenses possess many logical features in common with standard quantifiers in first-order logic. The considerations of the previous chapter concerning the ability of systems of tense logic to accommodate tenseless statements involving dates and the temporal order relations is really a further aspect of point (i). For the thesis maintained here is that so-called tenseless statements are in fact appropriately so-called, and are not statements of which we can ask at what time they hold true. Thus, not only are there statements like 'I am hotter than I was' which cannot be reduced to statements involving reference to just one time; there are also statements which are not true relative to any time in the sense that they ascribe a property to a thing or a relation amongst several things which applies to those items at a time. For example, the 'earlier than' relation does not hold between two moments at some time, nor does the identity relation hold between two individuals at a time, and nor does a dated statement indicating the time at which a predicate is correctly applied to one or more individuals hold true relative to some further time.

These considerations show that the various developments of Priorean tense logic we have considered do not yield a satisfactory analysis of temporal reference in English usage, and suggest that a first-order theory is more appropriate than modal logic in this area. Furthermore, the system of first-order logic adopted here

was designed to reflect a certain position outlined in the first chapter on the relation between tensed and tenseless statements and to give some explicit account of the role of and relation between the temporal order relations, dates and expressions indicating pastness, presentness and futurity. This last point perhaps concerns a more speculative, metaphysical issue, and some might take the view that considerations of idiomatic usage should not feature so prominently in arguments relevant to this matter. In this case I can only say that it should be borne in mind when assessing the relative merits of tense logic based on the supposed analogy with modal logic and the first-order theory presented here that it has been one of my concerns to develop a formal theory in which the position outlined in the first chapter could be represented, whereas it seems to me an opposing metaphysical view which I do not share with Prior is implicit in Priorean tense logic.

I shall conclude this final chapter with a brief review of some of the arguments that have been offered in favour of adopting modal logic in the formulation of tenses and against the use of first-order logic for this purpose. Some of these arguments have already been mentioned at various points throughout this essay, but have not yet been mentioned in this summary, and it will be useful to bring them together now.

Remaining for the moment with the more metaphysical aspects of tense logic, there is one issue we have already encountered but which raises a problem not yet mentioned. I refer to Prior's view outlined in the second section of the last chapter that moments of time and their order relations are merely constructions from tensed statements. Hans Kamp discusses some such view when considering the viability of using formulae of first-order logic containing variables ranging over times to express tensed statements. Such a formula, he says,

> . . .is meaningful only if times exist. And so this approach commits us to the assumption of the existence of time, in a sense which is prior to language. This commitment is a rather strong one, which some will be reluctant to make. It would be desirable to have a logical theory of that fragment of language in which moments are not mentioned explicitly . . . which is not based on the assumption that times exist — with the option of developing, on the basis of that theory, a concept of a moment, which could then be used for a semantical account of the part of language where moments are mentioned explicitly.[1]

The difficulty I find with this passage is that I cannot see that relegating moments to the semantic analysis in the metalanguage frees one from the commitment that someone who allows moment variables in the object language is said to have. Prior seems to have avoided this problem by not developing a semantics of his systems of tense logic, but not so with Kamp. According to his theory,[2] the set of integers constitute the model for the set of times in the semantics, and a date is a propositional constant d_n which is true at i if and only if $i=n$ (i.e., a date is said to

[1] J.A.W. Kamp, *Tense Logic and the Theory of Linear Order*, (Ph.D. thesis, University of California, Los Angeles, 1968), p. 4.

[2] Hans Kamp, 'Tense Logic with Dates', Unpublished Notes (1967).

be a proposition true at that instant only). Since times are employed in the semantics to explain how we are to understand the object language, I do not see how such a theory could be said not to carry an assumption of the existence of time which a theory involving moment variables in the object language is said to presuppose. Moreover, it is somewhat obscure in what sense explicit mention of dates has been removed from the object language on this account. For the indices on the propositions d_n serve not merely to differentiate such propositions, but also to correlate them with the corresponding integer in the semantics which is thought of as a moment of time.

From a less metaphysical perspective, a number of other reasons have been advanced in favour of adopting tense logic in the spirit of Prior rather than a first-order theory. There is a suggestion in the notes of Kamp referred to above that since tenses apparently do not make explicit reference to time, the object language in which logical analyses of tenses are formulated should also reflect this feature. Thus, of 'John cried' Kamp says it "is about John, but not about some (unspecified) moment of the past". It would be unfair to make too much of brief informal remarks in unpublished notes. But I do think it fair to say that much recent interest in tense logic has been motivated by the assumption that since no particular moment is explicitly referred to by the use of a tense (it is "unspecified"), the problem of a logic of tenses must be to show how expressions formulated in an object language devoid of moment terms can in fact refer to time. However, it seems to me that the frequent unspecified character of tenses is accounted for simply by saying that quantification over moments is involved, and not by the non-occurrence of moment variables in the object language. Moreover, sometimes the temporal reference expressed by the use of tenses is not unspecific; this is often the case, for example, when 'now' and 'then' occur. These are the so-called definite tenses which, apart from Kamp's 'now' operator, have no place in Priorean tense logic.

Again we turn to Kamp for two more points that have been raised against the viability of first-order logic in this area. Discussing the sentence 'Greece is a kingdom' (which he refers to by '(1)'), he says:

> In ordinary logic sentences are either true or false . . . according to the view embodied in ordinary model thoery, (1) is not really a sentence at all, since its truth value is not definite but fluctuates with time; on this view (1) is rather like a formula with one free variable, ranging over times, which is satisfied by a moment whenever, as we say, (1) "is true at" that moment.
>
> Indeed, we can symbolise sentences like (1) within first order logic, if we treat them as such formulae. But the theory implicit in such a treatment is unsatisfactory in various ways. In the first place it is rather odd that, as this theory suggests, most English sentences—among them the grammatically most central examples—should not really be sentences at all. I. e. the theory proposes a concept of 'sentence' which seems to have very little to do with the intuitive meaning of the word. In the second place the theory leads us to regard what appear to be n-place relations in English as (n+1)-place relations. Thus, the expression 'is a kingdom'—apparently a 1-place

predicate—has to be analysed as standing for a 2-place relation between countries and times.[3]

I hope it is now clear that it is not incumbent on a proponent of the formulation of tenses in first-order logic to adopt the view Kamp outlines as his first point here. Formulae with free moment variables are not interpreted as either tensed or tenseless on our account, so that the notion of 'sentence' at issue in our theory is not so much at variance with the intuitive meaning of the word as Kamp suggests. The second point appears rather spurious, and it seems to me the intuition behind our wanting to draw the distinction between predicates which ascribe properties to individuals relative to a time or times and those which do not is on a far firmer footing than the inclination to regard 'is a kingdom' as a one-place relation with no explicit mention of time. In any case, there is a clear sense on our theory in which 'is a kingdom' is a 1-place relation. For if we take this as a present tensed predicate (as I think Kamp would), then it would be expressed in the form $\Delta^{\alpha\alpha}P^1_j$, which is a 1-place predicate applying to individuals.

Another objection against the use of a first-order theory has been raised by Prior, who says that " 'now' is not the name of a date (it has the same meaning whenever it is used, but does not refer to the same date whenever it is used)".[4] Our theory treats 'now' in a similar manner to dates, for it would be an easy matter to remove the indexical quantifiers in favour of constants, replacing wffs of the kind $\Delta^{\alpha}A$ by $(n/\alpha)A$, where n is a moment constant, and similarly for the other indexical quantifiers. This seems to me appropriate, since 'now' picks out a definite moment just as a date does, and the discussion of quantifiers over individuals in tensed contexts bears out this contention. The fact that the reference of the word 'now' changes on different occasions of use has to be explained in terms of different interpretations **M** being appropriate at different times, but which differ only in so far as the points of reference are different.

As a final point, I want to consider the notion of a proposition with variable truth value that occurs in Priorean tense logic. For this purpose, consider

(5) Greece is a kingdom

which could have been used to make a true statement not so long ago, but before that time and now (January 1974) someone using (5) would have made a false statement. Because sentences like (5) can be used at different times to express propositions with different truth values, it is suggested that propositions of tense logic (at any rate, the wffs) should reflect this feature of sentences by themselves being assigned truth values which may differ from time to time. It was suggested towards the end of the previous chapter, however, that if we consider dated state-

[3] J.A.W. Kamp, *Tense Logic and the Theory of Linear Order*, (Ph. D. thesis, University of California, Los Angeles, 1968), pp. 3–4.

[4] *Past, Present and Future*, p. 41.

ments, it is by no means clear that adopting propositions with variable truth value is an appropriate step to take. For example, if we modify (5) so that we have

(6) On May 10, 1968 Greece is a kingdom,

this sentence could only have been used on May 10, 1968 to express a true proposition. To express the same state of affairs now, we would have to say 'On May 10, 1968 Greece was a kingdom', but this fact cannot be represented within the Priorean theory using either Prior's own account of dates or Mott's. However, this might not be enough to convince Kamp, for on his account of dates it is possible to express (6) in the form $P_m p$, where p stands for (5) and the operator P, where m is a date, is read as 'It was the case on m that _' and defined as follows:

$$P_m p = \text{df } P d_m \,\&\, T_m p$$

where d_m is the proposition true only at m, and T_m is defined by

$$T_m p =_{\text{df}} (d_m \,\&\, p) \vee P(d_m \,\&\, p) \vee F(d_m \,\&\, p)$$

Our other reasons for not favouring this account of dates apart, it might thus still be argued that (6) can be formulated in terms of propositions with variable truth values in a way which meets the present objection. (For this reason, Kamp's account of dates is much closer to the usual idiom than either of Prior's or Mott's systems.)

But (5) can be modified in another way which suits my purpose. First, note that according to the theory advocated here, the present tense is always formulated with an explicit present tense operator Δ^{α}, and that this operator is used to formulate occurrences of 'now' in English sentences. In accordance with this procedure, it seems to me that it should always be possible to introduce the word 'now' in an appropriate way in present tense statements without change of sense. In particular, though it is not necessary to use the word 'now' to convey what would be conveyed by using (5), the sentence

(7) Greece is now a kingdom

is nevertheless synonymous with (5). I think both Prior and Kamp would agree with this. However, (7) behaves very much like (6) does on my view, for if someone had used (7) in the past to convey to me an existing state of affairs, I would now report what was said to me by using the sentence 'Greece was (then) a kingdom'. Of course, it is true of both (5) and (7) that on different occasions they can be used to make statements whose truth values differ. But it seems to me that an interesting point is bypassed in Priorean tense logic by imitating this feature. For if we want to consider the state of affairs reported by the use of (7) in the past, we cannot now refer to this state of affairs by using a sentence which is simply a

past tense modification of (7) in the Priorean manner, namely 'It was the case that Greece is now a kingdom', where (7) remains intact as part of this new sentence. For 'now' refers to the moment of speaking, and since this is different on the two occasions of speaking, we cannot use the same sentence (7) as part of another sentence used at a later time to report the same state of affairs that would have been reported when (7) could have been used to make a true statement. Our point of view changes, not the state of affairs under discussion, and this is what is reflected by the use of different sentences on different occasions to refer to the same state of affairs.

A simple picture of what is going on here is to consider that on the two occasions of speaking, the models which provide the interpretations for the sentences used on those occasions are the same except for the times denoted by the reference points. The present moment of the first model becomes the point of reference in the past for the second model. A *proposition* might then be viewed as a triple comprising a formula B, an interpretation $\mathbf{M}$ for the language of which B is a member, and a sequence $\mathbf{s}$ comprising members of $\mathbf{D}$ and $\mathbf{T}$ in $\mathbf{M}$ as defined in Chapter III. Let us call a proposition $(B,\mathbf{M},\mathbf{s})$ *tensed* when B is a tensed statement as defined in Chapter IV, and if B is a tenseless statement, a *tenseless proposition*. Whether $\mathbf{M}$ satisfies B at $\mathbf{s}$ depends upon the reference points in $\mathbf{M}$ if $(B,\mathbf{M},\mathbf{s})$ is a tensed proposition, but not if $(B,\mathbf{M},\mathbf{s})$ is a tenseless proposition, so we can say that a tensed proposition reflects the point of view of whoever expresses it, whereas a tenseless proposition does not. On this picture, two formulae B and B' can be said to represent sentence types such that B can be used to express a proposition $(B,\mathbf{M},\mathbf{s})$ which describes the same state of affairs, from a different point of view, as the proposition $(B',\mathbf{M}',\mathbf{s}')$ expressed by the use of the sentence type B', provided

(i) $\mathbf{s} = \mathbf{s}'$;

(ii) $\mathbf{M}$ differs from $\mathbf{M}'$ only in that the reference points $\mathbf{p},\mathbf{n},\mathbf{f} \in \mathbf{M}$ are not the same members of $\mathbf{T} \in \mathbf{M},\mathbf{M}'$ as the corresponding reference points $\mathbf{p}'$, $\mathbf{n}'$, $\mathbf{f}' \in \mathbf{M}'$, and $\mathbf{n}' = \mathbf{p}$; and

(iii) B is like B' except that all occurrences of Π^{α} in B' are replaced by $\Pi^{\beta\beta}P^{\alpha}$ in B, where β is not free in B, all occurrences of Δ^{α} in B' are replaced by Π^{α} in B, and all occurrences of Γ^{α} in B' are replaced by $\Pi^{\gamma\gamma}F^{\alpha}$ in B, where γ does not occur free in B.

It does not follow from this definition that $(B',\mathbf{M}',\mathbf{s}')$ describes the same state of affairs, from a different point of view, as $(B,\mathbf{M},\mathbf{s})$, so this is certainly not an equivalence relation we have defined between propositions. This consequence seems to me appropriate, for it reflects the fact that once past, a given temporal perspective cannot be acquired again. (The situation would not be the same for spatial indexicals analogous to the present system, except that we have only two indexical quantifiers Σ^{α} and Θ^{α} for 'here' and 'there', respectively. The corresponding reference points $\mathbf{h}$ and $\mathbf{t}$ occurring in an interpretation $\mathbf{M}$ of a proposi-

tion (B,**M**,**s**) would then be interchanged, in clause (ii) of the corresponding definition, with $\mathbf{h}'$ and $\mathbf{t}'$ of $\mathbf{M}'\varepsilon(B',\mathbf{M}',s')$, and Σ^{α} and Θ^{α} would be interchanged in B and B' in the corresponding clause (iii).)

Perhaps some of the restrictions on the similarity required between **M** and $\mathbf{M}'$ could be relaxed, and the substitution of tenses in B' to obtain B made sensitive to a more detailed specification of the relation between the points of view. But this is sufficient to show that our first-order theory yields an account of the relation between a sentence type and the proposition expressed by its use on a particular occasion which provides some insight into an important feature of tenses, namely that different sentences are used on different occasions to report the same state of affairs.

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